4

First Law of Thermodynamics

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4.1. INTERNAL ENERGY

It is the heat energy stored in a gas. If a certain amount of heat is supplied to a gas the result is that temperature of gas may increase or volume of gas may increase thereby doing some external work or both temperature and volume may increase ; but it will be decided by the conditions under which the gas is supplied heat. *If during heating of the gas the temperature increases its internal energy will also increase.*

Joule's law of internal energy states that internal energy of a perfect gas is a *function of temperature only.* In other words, internal energy of a gas is dependent on the temperature change only and is not affected by the change in pressure and volume.

We do not know how to find the absolute quantity of internal energy in any substance ; however, what is needed in engineering is the change of internal energy (ΔU).

4.2. LAW OF CONSERVATION OF ENERGY

In the early part of nineteenth century the scientists developed the concept of energy and hypothesis that it can be neither created nor destroyed ; this came to be known as the *law of the conservation of energy*. The first law of thermodynamics is merely one statement of this general law/principle with particular reference to heat energy and mechanical energy *i.e.,* work.

4.3. FIRST LAW OF THERMODYNAMICS

It is observed that when a system is made to undergo a complete cycle then net work is done *on* or *by* the system. Consider a cycle in which net work is done by the system. Since energy cannot be created, this mechanical energy must have been supplied from some source of energy. Now the system has been returned to its initial state : Therefore, its *intrinsic* energy is unchanged, and hence the mechanical energy has not been provided by the system itself. The only other energy involved in the cycle is the heat which was supplied and rejected in various processes. Hence, by the law of conservation of energy, the net work done by the system is equal to the net heat supplied to the system. The First Law of Thermodynamics can, therefore, be stated as follows :

"When a system undergoes a thermodynamic cycle then the net heat supplied to the system from the surroundings is equal to net work done by the system on its surroundings.

or $$\oint dQ = \oint dW$$

where $\oint$ represents the sum for a complete cycle.

The first law of Thermodynamics *cannot be proved analytically, but experimental evidence has repeatedly confirmed its validity,* and since no phenomenon has been shown to contradict it, the first law is accepted as a *law of nature.* It may be remarked that no restriction was imposed which limited the application of first law to reversible energy transformation. Hence the first law applies to reversible as well as irreversible transformations : For non-cyclic process, a more general formulation of first law of thermodynamics is required. A new concept which involves a term called *internal energy* fulfills this need.

— The First Law of Thermodynamics may also be stated as follows :

"Heat and work are mutually convertible but since energy can neither be created nor destroyed, the total energy associated with an energy conversion remains constant".

Or

— **"No machine can produce energy without corresponding expenditure of energy, *i.e.,* it is impossible to construct a perpetual motion machine of first kind".**

Fig. 4.1 shows the experiment for checking first law of thermodynamics.

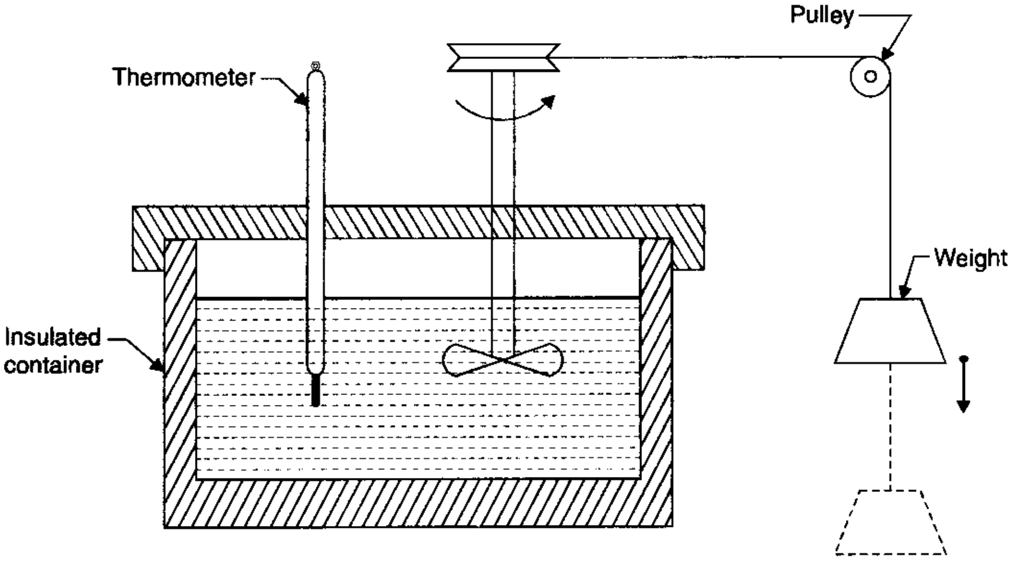


Fig. 4.1. Heat and work.

The work input to the paddle wheel is measured by the fall of weight, while the corresponding temperature rise of liquid in the insulated container is measured by the thermometer. It is already known to us from experiments on heat transfer that temperature rise can also be produced by heat transfer. The experiments show : (*i*) A definite quantity of work is always required to accomplish the same temperature rise obtained with a unit amount of heat. (*ii*) Regardless of whether the temperature of liquid is raised by work transfer or heat transfer, the liquid can be returned by heat transfer in opposite direction to the identical state from which it started. The above results lead to the inference that *work and heat* are different forms of something more general, which is called *energy*.

— It can be stated as an invariable experience that whenever a physical system passes through a complete cycle the algebraic sum of the work transfers during the cycle $\oint dW$ bears a definite ratio to the algebraic sum of the heat transfers during the cycle, $\oint dQ$. This may be expressed by the equation,

$$\oint dW = J \oint dQ \qquad \ldots(4.1)$$

where J is the proportionality constant and is known as *Mechanical Equivalent of heat*. In S.I. units its value is unity, *i.e.*, 1 Nm/J.

4.4. APPLICATION OF FIRST LAW TO A PROCESS

When a process is executed by a system, the *change in stored energy of the system is numerically equal to the net heat interactions minus the net work interaction during the process.*

$$\therefore \qquad E_2 - E_1 = Q - W$$

$$\therefore \qquad \Delta E = Q - W \qquad [\text{or } Q = \Delta E + W]$$

or

$$\int_1^2 d(Q - W) = \Delta E = E_2 - E_1 \qquad \ldots(4.2)$$

where E represents the *total internal energy*.

If the electric, magnetic and chemical energies are *absent* and changes in potential and kinetic energy for a *closed system* are neglected, the above equation can be written as

$$\int_1^2 d(Q - W) = \Delta U = U_2 - U_1 \qquad \ldots(4.3)$$

$$\therefore \qquad Q - W = \Delta U = U_2 - U_1 \qquad \ldots(4.4)$$

Generally, when heat is added to a system its temperature rises and external work is performed due to increase in volume of the system. *The rise in temperature is an indication of increase of internal energy.*

Heat *added* to the system will be considered as *positive* and the heat *removed* or *rejected*, from the system, as *negative*.

4.5. ENERGY—A PROPERTY OF SYSTEM

Consider a system which changes its state from state 1 to state 2 by following the path L, and returns from state 2 to state 1 by following the path M (Fig. 4.2). So the system undergoes a cycle. Writing the first law for path L

$$Q_L = \Delta E_L + W_L \qquad \ldots(4.5)$$

and for path M

$$Q_M = \Delta E_M + W_M \qquad \ldots(4.6)$$

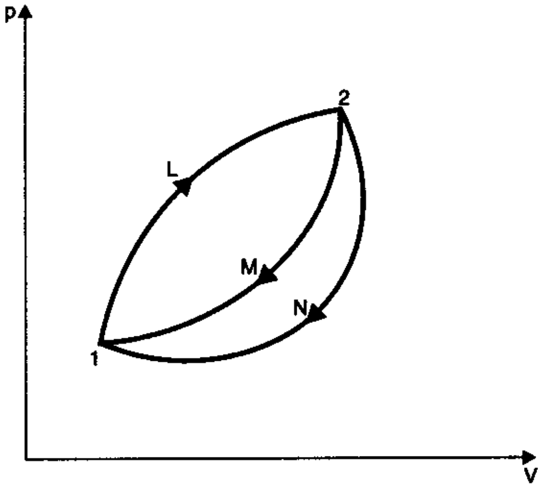


Fig. 4.2. Energy—a property of system.

The processes L and M together constitute a cycle, for which

$$\oint dW = \oint dQ$$

$$W_L + W_M = Q_L + Q_M$$

or
$$Q_L - W_L = W_M - Q_M \quad ...(4.7)$$

From equations (4.5), (4.6) and (4.7), it yields

$$\Delta E_L = - \Delta E_M \quad ...(4.8)$$

Similarly, had the system returned from state 2 to state 1 by following the path N instead of path M

$$\Delta E_L = - \Delta E_N \quad ...(4.9)$$

From equations (4.8) and (4.9),

$$\Delta E_M = \Delta E_N \quad ...(4.10)$$

Thus, it is seen that the *change in energy between two states of a system is the same, whatever path the system may follow in undergoing that change of state.* If some arbitrary value of energy is assigned to state 2, the value of energy at state 1 is fixed independent of the path the system follows. Therefore, *energy has a definite value for every state of the system.* Hence, it is *a point function and a property of the system.*

4.6. PERPETUAL MOTION MACHINE OF THE FIRST KIND—PMM 1

The first law of thermodynamics states the general principle of the conservation of energy. Energy is neither created nor destroyed, but only gets transformed from one form to another. *There can be no machine which would continuously supply mechanical work without some form of energy disappearing simultaneously* (Fig. 4.3). Such a *fictitious machine* is called a ***perpetual motion machine of the first kind,*** or in brief, PMM 1. A PMM 1 is thus ***impossible.***

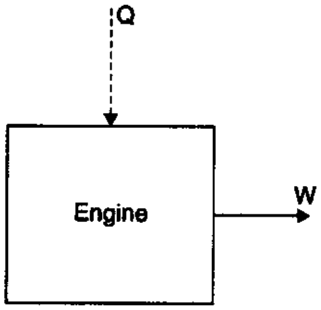


Fig. 4.3. A PPM 1.

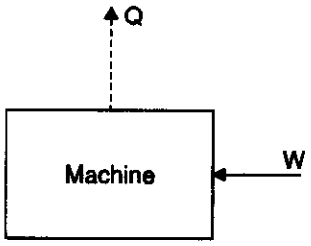


Fig. 4.4. The converse of PMM 1.

- The converse of the above statement is also true, *i.e.*, there can be no machine which would continuously consume work without some other form of energy appearing simultaneously (Fig. 4.4).

4.7. ENERGY OF AN ISOLATED SYSTEM

An isolated system is one in which there is no interaction of the system with the surroundings.

For an isolated system,

$$dQ = 0,\ dW = 0$$

The first law of thermodynamics gives

$$dE = 0$$

or

$$E = \text{constant}$$

The energy of an isolated system is always constant.

4.8. THE PERFECT GAS

4.8.1. The Characteristic Equation of State

- At temperatures that are considerably in excess of critical temperature of a fluid, and also at very low pressure, the vapour of fluid tends to obey the equation

$$\frac{pv}{T} = \text{constant} = R$$

In practice, no gas obeys this law rigidly, but many gases tend towards it.

An imaginary ideal gas which obeys this law is called a *perfect gas*, and the equation $\frac{pv}{T} = R$, is called the *characteristic equation of a state of a perfect gas.* The constant R is called the *gas constant*. Each perfect gas has a different gas constant.

Units of R are Nm/kg K or kJ/kg K.

Usually, the characteristic equation is written as

$$pv = RT \qquad ...(4.11)$$

or for m kg, occupying V m^3

$$pV = mRT \qquad ...(4.12)$$

- The characteristic equation in *another form*, can be derived by using kilogram-mole as a unit.

The *kilogram-mole* is defined as a quantity of a gas equivalent to M kg of the gas, where M is the molecular weight of the gas (*e.g.*, since the molecular weight of oxygen is 32, then 1 kg mole of oxygen is equivalent to 32 kg of oxygen).

As per definition of the kilogram-mole, for m kg of a gas, we have

$$m = nM \qquad \text{...(4.13)}$$

where n = number of moles.

Note. Since the standard of mass is the kg, kilogram-mole will be written simply as mole.

Substituting for m from eqn. (4.13) in eqn. (4.12) gives

$$pV = nMRT$$

or

$$MR = \frac{pV}{nT}$$

According to *Avogadro's hypothesis* the volume of 1 mole of any gas is the same as the volume of 1 mole of any other gas, when the gases are at the same temperature and pressure. Therefore, $\frac{V}{n}$ is the same for all gases at the same value of p and T. That is the quantity $\frac{pV}{nT}$ is a *constant* for all gases. This constant is called *universal gas constant*, and is given the symbol, R_0.

i.e.,

$$MR = R_0 = \frac{pV}{nT}$$

or

$$pV = nR_0T \qquad \text{...(4.14)}$$

Since $MR = R_0$, then

$$R = \frac{R_0}{M} \qquad \text{...(4.15)}$$

It has been found experimentally that the volume of 1 mole of any perfect gas at 1 bar and 0°C is approximately 22.71 m^3.

Therefore from eqn. (4.14),

$$R_0 = \frac{pV}{nT} = \frac{1 \times 10^5 \times 22.71}{1 \times 273.15}$$

$$= 8314.3 \text{ Nm/mole K}$$

Using eqn. (4.15), the gas constant for any gas can be found when the molecular weight is known.

Example. For oxygen which has a molecular weight of 32, the gas constant

$$R = \frac{R_0}{M} = \frac{8314}{32} = 259.8 \text{ Nm/kg K.}$$

4.8.2. Specific Heats

The **specific heat** of a solid or liquid is usually defined as the *heat required to raise unit mass through one degree temperature rise.*

For small quantities, we have

$$dQ = mcdT$$

where m = mass,

c = specific heat, and

dT = temperature rise.

For a gas there are an infinite number of ways in which heat may be added between any two temperatures, and hence a *gas could have an infinite number of specific heats.* However, only two specific heats for gases are defined.

Specific heat at constant volume, c_v

and, *Specific heat at constant pressure,* c_p.

We have

$dQ = m\ c_p\ dT$ For a reversible non-flow process at *constant pressure* ...(4.16)

and, $dQ = m\ c_v\ dT$ For a reversible non-flow process at *constant volume* ...(4.17)

The values of c_p and c_v, for a perfect gas, are constant for any one gas at all pressures and temperatures. Hence, integrating eqns. (4.16) and (4.17), we have

Flow of heat in a reversible constant pressure process

$$= mc_p\ (T_2 - T_1) \quad ...(4.18)$$

Flow of heat in a reversible constant volume process

$$= mc_v\ (T_2 - T_1) \quad ...(4.19)$$

In case of *real gases,* c_p and c_v *vary with temperature,* but a suitable *average value* may be used for most practical purposes.

4.8.3. Joule's Law

Joule's law states as follows :

"The internal energy of a perfect gas is a function of the absolute temperature only."

i.e., $u = f(T)$

To evaluate this function let 1 kg of a perfect gas be heated at constant volume.

According to non-flow energy equation,

$$dQ = du + dW$$

$dW = 0$, since volume remains constant

$$\therefore \quad dQ = du$$

At constant volume for a perfect gas, from eqn. (4.17), for 1 kg

$$dQ = c_v dT$$

$$\therefore \quad dQ = du = c_v dT$$

and integrating $u = c_v\ T + K$, K being constant.

According to Joule's law $u = f(T)$, which means that internal energy varies linearly with absolute temperature. Internal energy can be made zero at any arbitrary reference temperature. For a perfect gas it can be assumed that $u = 0$ when $T = 0$, hence constant K is zero.

i.e., Internal energy, $u = c_v\ T$ for a perfect gas ...(4.20)

or For mass m, of a perfect gas

Internal energy, $U = mc_v\ T$ ...(4.21)

For a perfect gas, in any process between states 1 and 2, we have from Eqn. (4.21)

Gain in internal energy,

$$U_2 - U_1 = mc_v\ (T_2 - T_1) \quad ...(4.22)$$

Eqn. (4.22) gives the gains of internal energy for a perfect gas between two states *for any process, reversible* or *irreversible.*

4.8.4. Relationship Between Two Specific Heats

Consider a perfect gas being heated at constant pressure from T_1 to T_2.

According to non-flow equation,

$$Q = (U_2 - U_1) + W$$

Also for a perfect gas,

$$U_2 - U_1 = mc_v\ (T_2 - T_1)$$

$$Q = mc_v\ (T_2 - T_1) + W$$

In a constant pressure process, the work done by the fluid,

$$W = p(V_2 - V_1)$$

$$= mR(T_2 - T_1) \qquad \begin{bmatrix} \because \; p_1V_1 = mRT_1 \\ p_2V_2 = mRT_2 \\ p_1 = p_2 = p \text{ in this case} \end{bmatrix}$$

On substituting

$$Q = mc_v\,(T_2 - T_1) + mR\,(T_2 - T_1) = m(c_v + R)\,(T_2 - T_1)$$

But for a constant pressure process,

$$Q = mc_p\,(T_2 - T_1)$$

By equating the two expressions, we have

$$m(c_v + R)(T_2 - T_1) = mc_p(T_2 - T_1)$$

$$\therefore \qquad c_v + R = c_p$$

or

$$c_p - c_v = R \qquad \text{...(4.23)}$$

Dividing both sides by c_v, we get

$$\frac{c_p}{c_v} - 1 = \frac{R}{c_v}$$

$$\therefore \qquad c_v = \frac{R}{\gamma - 1} \qquad \text{...[4.23 (a)]}$$

(where $\gamma = c_p/c_v$)

Similarly, dividing both sides by c_p, we get

$$c_p = \frac{\gamma R}{\gamma - 1} \qquad \text{...[4.23 (b)]}$$

$$\begin{bmatrix} \text{In M.K.S. units} : c_p - c_v = \dfrac{R}{J};\; c_v = \dfrac{R}{J(\gamma - 1)},\; c_p = \dfrac{\gamma R}{(\gamma - 1)J} \\ \text{In SI units the value of } J \text{ is unity.} \end{bmatrix}$$

4.8.5. Enthalpy

- One of the fundamental quantities which occur invariably in thermodynamics is the sum of internal energy (u) and pressure volume product (pv). This sum is called **Enthalpy** (h).

  i.e., $$h = u + pv \qquad \text{...(4.24)}$$

- The enthalpy of a fluid is the property of the fluid, since it consists of the sum of a property and the product of the two properties. Since enthalpy is a property like internal energy, pressure, specific volume and temperature, it can be introduced into any problem whether the process is a flow or a non-flow process.

The total enthalpy of mass, m, of a fluid can be

$$H = U + pV, \text{ where } H = mh.$$

For a **perfect gas,**

Referring equation (4.24),

$$h = u + pv$$

$$= c_vT + RT \qquad [\because \; pv = RT]$$
$$= (c_v + R)T$$
$$= c_pT \qquad [\because \; c_p = c_v + R]$$

i.e., $h = c_pT$

and $H = mc_pT.$

(Note that, since it has been assumed that $u = 0$ at $T = 0$, then $h = 0$ at $T = 0$).

4.8.6. Ratio of Specific Heats

The ratio of specific heat at constant pressure to the specific heat at constant volume is given the symbol γ (gamma).

i.e., $$\gamma = \frac{c_p}{c_v} \qquad \text{...(4.25)}$$

Since $c_p = c_v + R$, it is clear that c_p must be *greater* than c_v for any perfect gas. It follows, therefore, that the ratio, $\frac{c_p}{c_v} = \gamma$ is *always greater than unity.*

In general, the approximate values of γ are as follows :

For *monoatomic* gases such as *argon, helium* = 1.6.

For *diatomic* gases such as *carbon monoxide, hydrogen, nitrogen* and *oxygen* = 1.4.

For triatomic gases such as *carbon dioxide* and *sulphur dioxide* = 1.3.

For some hydro-carbons the value of γ is quite low.

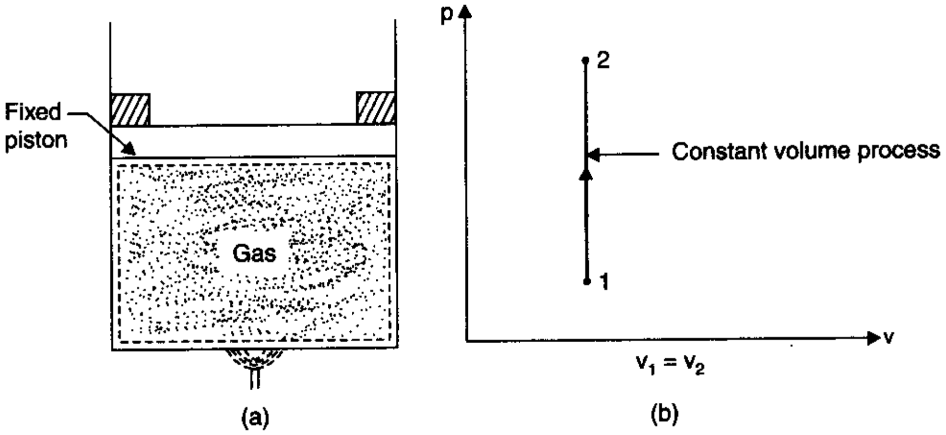
[*e.g.*, for ethane γ = 1.22, and for isobutane γ = 1.11]

4.9. APPLICATION OF FIRST LAW OF THERMODYNAMICS TO NON-FLOW OR CLOSED SYSTEM

1. Reversible Constant Volume (or Isochoric) Process (v = constant) :

In a constant volume process the working substance is contained in a rigid vessel, hence the boundaries of the system are immovable and no work can be done on or by the system, other than paddle-wheel work input. It will be assumed that '*constant volume*' implies zero work unless stated otherwise.

Fig. 4.5 shows the system and states before and after the heat addition at constant volume.



(a) (b)

Fig. 4.5. Reversible constant volume process.

Considering mass of the working substance *unity* and applying first law of thermodynamics to the process

$$Q = (u_2 - u_1) + W \qquad \text{...(4.26)}$$

The work done $W = \int_1^2 p\,dv = 0$ as $dv = 0$.

$$\therefore \quad Q = (u_2 - u_1) = c_v(T_2 - T_1) \qquad \text{...[4.27 (a)]}$$

where c_v = Specific heat at constant volume.

For mass, m, of working substance

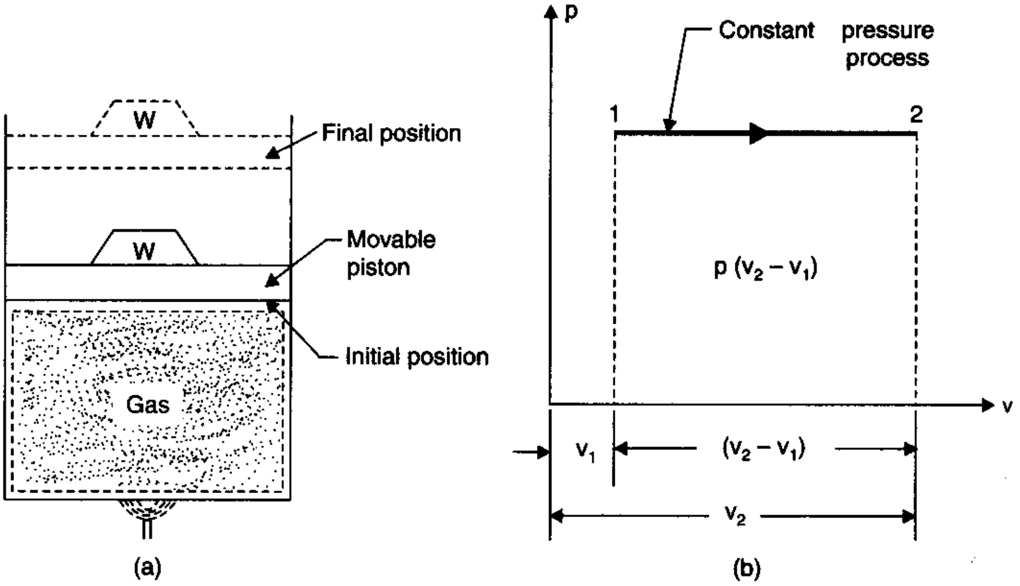
$$Q = U_2 - U_1 = mc_v(T_2 - T_1) \qquad \text{...[4.27 (b)]}$$

$$[\because \quad mu = U]$$

2. Reversible Constant Pressure (or Isobaric) Process (p = constant).

It can be seen from Fig. 4.5 (*b*) that when the boundary of the system is *inflexible* as in a constant volume process, then the pressure rises when heat is supplied. Hence for a constant pressure process, the boundary must move against an external resistance as heat is supplied ; for instance a gas [Fig. 4.6 (*a*)] in a cylinder behind a piston can be made to undergo a constant pressure process. Since the *piston is pushed through a certain distance* by the force exerted by the gas, then the work is done by the gas on its surroundings.

Fig. 4.6 shows the system and states before and after the heat addition at constant pressure.



(a) (b)

Fig. 4.6. Reversible constant pressure process.

Considering unit mass of working substance and applying first law of thermodynamics to the process

$$Q = (u_2 - u_1) + W$$

The work done, $W = \int_1^2 p\,dv = p(v_2 - v_1)$

$$\therefore \quad Q = (u_2 - u_1) + p(v_2 - v_1) = u_2 - u_1 + pv_2 - pv_1$$

$$= (u_2 + pv_2) - (u_1 + pv_1) = h_2 - h_1 \qquad [\because \; h = u + pv]$$

or
$$Q = h_2 - h_1 = c_p\,(T_2 - T_1) \qquad \text{...(4.28)}$$

where h = Enthalpy (specific), and

c_p = Specific heat at constant pressure.

For mass, m, of working substance

$$Q = H_2 - H_1 = mc_p\,(T_2 - T_1) \qquad \text{...[4.28 (a)]}$$

$$[\because \; mh = H]$$

3. Reversible Temperature (or Isothermal) Process (pv = constant, T = constant) :

A *process at a constant temperature is called an isothermal process.* When a working substance in a cylinder behind a piston expands from a high pressure to a low pressure there is a tendency for the temperature to fall. In an isothermal expansion heat must be added continuously in order to keep the temperature at the initial value. Similarly in an isothermal compression heat must be removed from the working substance continuously during the process.

Fig. 4.7 shows the system and states before and after the heat addition at constant temperature.

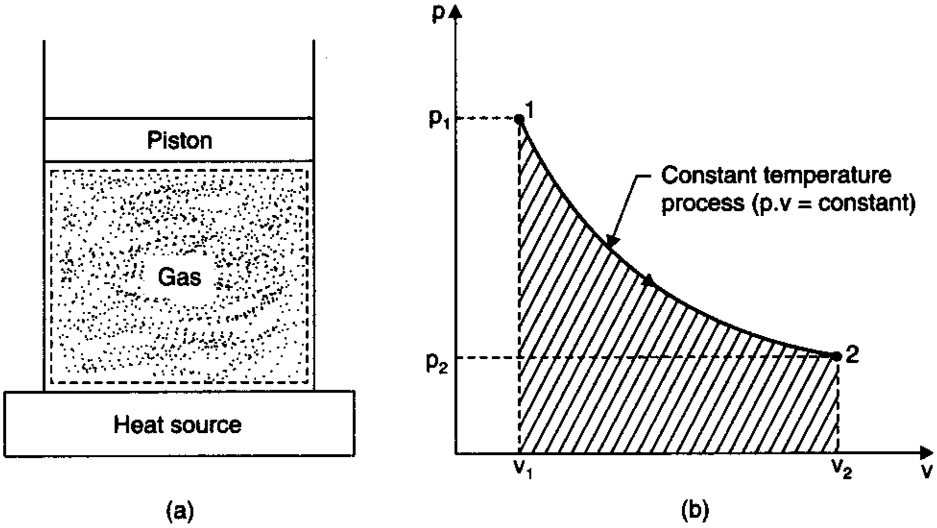


Fig. 4.7. Reversible isothermal process.

Considering unit mass of working substance and applying first law to the process

$$Q = (u_2 - u_1) + W$$
$$= c_v\,(T_2 - T_1) + W$$
$$= 0 + W \qquad [\because \; T_2 = T_1]$$

The work done, $W = \int_1^2 p\,dv$

In this case pv = constant or $p = \frac{C}{v}$ (where C = constant)

$$\therefore \qquad W = \int_{v_1}^{v_2} C\frac{dv}{v} = C\left[\log_e v\right]_{v_1}^{v_2} = C\log_e \frac{v_2}{v_1}$$

The constant C can either be written as p_1v_1 or as p_2v_2, since

$$p_1v_1 = p_2v_2 = \text{constant, } C$$

i.e.,
$$W = p_1v_1 \log_e \frac{v_2}{v_1} \text{ per unit mass of working substance}$$

or
$$W = p_2v_2 \log_e \frac{v_2}{v_1} \text{ per unit mass of working substance}$$

$\therefore$
$$Q = W = p_1v_1 \log_e \frac{v_2}{v_1} \qquad \text{...(4.29)}$$

For mass, m, of the working substance

$$Q = p_1V_1 \log_e \frac{V_2}{V_1} \qquad \text{...[4.29 (a)]}$$

or
$$Q = p_1V_1 \log_e \frac{p_1}{p_2} \qquad \left[\because \frac{V_2}{V_1} = \frac{p_1}{p_2}\right] \qquad \text{...[4.29 (b)]}$$

4. Reversible Adiabatic Process (pv^γ = constant) :

An **adiabatic process** *is one in which no heat is transferred to or from the fluid during the process.* Such a process can be reversible or irreversible. The reversible adiabatic non-flow process will be considered in this section.

Considering unit mass of working substance and applying first law to the process

$$Q = (u_2 - u_1) + W$$
$$O = (u_2 - u_1) + W$$

or
$$W = (u_1 - u_2) \text{ for any adiabatic process} \qquad \text{...(4.30)}$$

Eqn. (4.30) is true for an adiabatic process whether the process is reversible or not. In an adiabatic expansion, the work done W by the fluid is at the expense of a reduction in the internal energy of the fluid. Similarly in an adiabatic compression process all the work done on the fluid goes to increase the internal energy of the fluid.

For an adiabatic process to take place, perfect thermal insulation for the system must be available.

To derive the law pv$^\gamma$ = constant :

To obtain a law relating p and v for a reversible adiabatic process let us consider the non-flow energy equation in differential form,

$$dQ = du + dW$$

For a *reversible process*

$$dW = pdv$$

$\therefore$
$$dQ = du + pdv = 0$$

(Since for an adiabatic process $Q = 0$)

Also for a perfect gas

$$pv = RT \text{ or } p = \frac{RT}{v}$$

Hence substituting,

$$du + \frac{RTdv}{v} = 0$$

Also
$$u = c_vT \text{ or } du = c_vdT$$

$\therefore$
$$c_vdT + \frac{RTdv}{v} = 0$$

Dividing both sides by T, we get

$$c_v \frac{dT}{T} + \frac{Rdv}{v} = 0$$

Integrating

$$c_v \log_e T + R \log_e v = \text{constant}$$

Substituting $T = \frac{pv}{R}$

$$c_v \log_e \frac{pv}{R} + R \log_e v = \text{constant}$$

Dividing throughout both sides by c_v

$$\log_e \frac{pv}{R} + \frac{R}{c_v} \cdot \log_e v = \text{constant}$$

Again $$c_v = \frac{R}{(\gamma - 1)} \quad \text{or} \quad \frac{R}{c_v} = \gamma - 1$$

Hence substituting

$$\log_e \frac{pv}{R} + (\gamma - 1) \log_e v = \text{constant}$$

$$\therefore \quad \log_e \frac{pv}{R} + \log_e v^{\gamma - 1} = \text{constant}$$

$$\log_e \frac{pv \times v^{\gamma - 1}}{R} = \text{constant}$$

i.e., $$\log_e \frac{pv^\gamma}{R} = \text{constant}$$

i.e., $$\frac{pv^\gamma}{R} = e^{\text{constant}} = \text{constant}$$

or $$pv^\gamma = \text{constant} \quad \ldots(4.31)$$

Expression for work W :

A reversible adiabatic process for a perfect gas is shown on a p-v diagram in Fig. 4.8 (b).

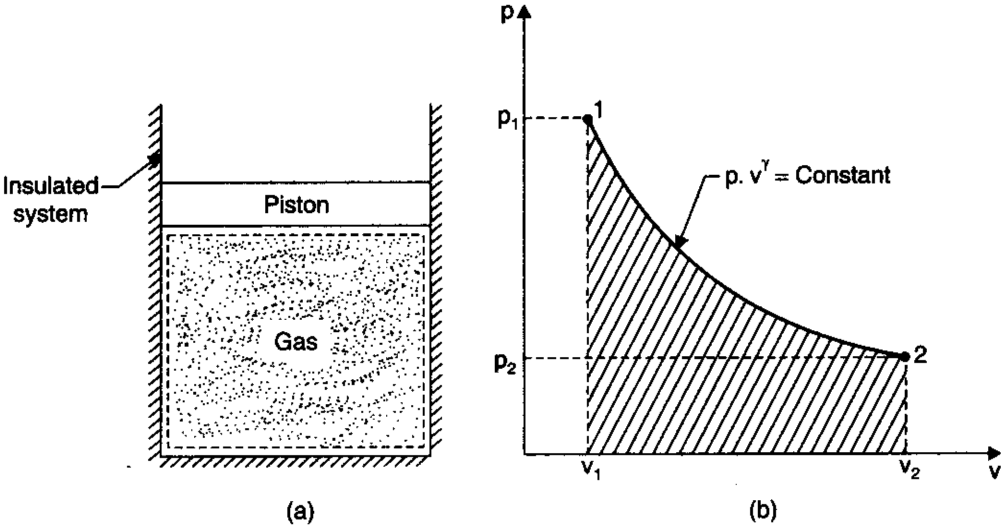


Fig. 4.8. Reversible adiabatic process.

The work done is given by the shaded area, and this area can be evaluated by integration.

i.e.,
$$W = \int_{v_1}^{v_2} p\, dv$$

Therefore, since pv^γ = constant, C, then

$$W = \int_{v_1}^{v_2} C \frac{dv}{v^\gamma} \qquad \left[\because\ p = \frac{C}{v^\gamma}\right]$$

i.e.,
$$W = C\int_{v_1}^{v_2} \frac{dv}{v^\gamma} = C \left| \frac{v^{-\gamma+1}}{-\gamma+1} \right|_{v_1}^{v_2}$$

$$= C\left(\frac{v_2^{-\gamma+1} - v_1^{-\gamma+1}}{1-\gamma}\right) = C\left(\frac{v_1^{-\gamma+1} - v_2^{-\gamma+1}}{\gamma - 1}\right)$$

The constant in this equation can be written as $p_1 v_1^\gamma$ or as $p_2 v_2^\gamma$. Hence,

$$W = \frac{p_1 v_1^\gamma v_1^{-\gamma+1} - p_2 v_2^\gamma v_2^{-\gamma+1}}{\gamma - 1} = \frac{p_1 v_1 - p_2 v_2}{\gamma - 1}$$

i.e.,
$$W = \frac{p_1 v_1 - p_2 v_2}{\gamma - 1} \qquad ...(4.32)$$

or
$$W = \frac{R(T_1 - T_2)}{\gamma - 1} \qquad ...(4.33)$$

Relationship between T and v, and T and p :

By using equation $pv = RT$, the relationship between T and v, and T and p, may by derived as follows :

i.e.,
$$pv = RT$$

$$\therefore \quad p = \frac{RT}{v}$$

Putting this value in the equation pv^γ = constant

$$\frac{RT}{v} . v^\gamma = \text{constant}$$

i.e.,
$$Tv^{\gamma-1} = \text{constant} \qquad ...(4.34)$$

Also $v = \dfrac{RT}{p}$; hence substituting in equation pv^γ = constant

$$p\left(\frac{RT}{p}\right)^\gamma = \text{constant}$$

$$\therefore \quad \frac{T^\gamma}{p^{\gamma-1}} = \text{constant}$$

or
$$\frac{T}{(p)^{\frac{\gamma-1}{\gamma}}} = \text{constant} \qquad ...(4.35)$$

Therefore, for a reversible adiabatic process for a perfect gas between states 1 and 2, we can write :

From Eqn. (4.31),

$$p_1v_1^{\gamma} = p_2v_2^{\gamma} \quad \text{or} \quad \frac{p_2}{p_1} = \left(\frac{v_1}{v_2}\right)^{\gamma} \qquad \text{...(4.36)}$$

From Eqn. (4.34),

$$T_1v_1^{\gamma-1} = T_2v_2^{\gamma-1} \quad \text{or} \quad \frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{\gamma-1} \qquad \text{...(4.37)}$$

From Eqn. (4.35),

$$\frac{T_1}{(p_1)^{\frac{\gamma-1}{\gamma}}} = \frac{T_2}{(p_2)^{\frac{\gamma-1}{\gamma}}} \quad \text{or} \quad \frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} \qquad \text{...(4.38)}$$

From eqn. (4.30), the work done in an adiabatic process per kg of gas is given by $W = (u_1 - u_2)$. The gain in internal energy of a perfect gas is given by equation :

$$u_2 - u_1 = c_v\,(T_2 - T_1) \qquad \text{(for 1 kg)}$$

$$\therefore \quad W = c_v\,(T_1 - T_2)$$

Also, we know that

$$c_v = \frac{R}{\gamma - 1}$$

Hence substituting, we get

$$W = \frac{R\,(T_1 - T_2)}{\gamma - 1}$$

Using equation, $pv = RT$

$$W = \frac{p_1v_1 - p_2v_2}{\gamma - 1}$$

This is the same expression obtained before as eqn. (4.32).

5. Polytropic Reversible Process (pv^n = constant) :

It is found that many processes in practice approximate to a reversible law of form pv^n = constant, where n is a constant. Both vapours and perfect gases obey this type of law closely in many non-flow processes. Such processes are *internally reversible*.

We know that for any reversible process,

$$W = \int p\,dv$$

For a process in pv^n = constant, we have

$$p = \frac{C}{v^n}, \text{ where } C \text{ is a constant}$$

$$\therefore \quad W = C\int_{v_1}^{v_2} \frac{dv}{v^n} = C\left|\frac{v^{-n+1}}{-n+1}\right| = C\left(\frac{v_2^{-n+1} - v_1^{-n+1}}{-n+1}\right)$$

i.e.,

$$W = C\left(\frac{v_1^{-n+1} - v_2^{-n+1}}{n-1}\right) = \frac{p_1v_1^nv_1^{-n+1} - p_2v_2^nv_2^{-n+1}}{n-1}$$

(since the constant C, can be written as $p_1v_1^n$ or as $p_2v_2^n$)

i.e., Work done, $W = \dfrac{p_1v_1 - p_2v_2}{n-1}$ ...(4.39)

or $W = \dfrac{R(T_1 - T_2)}{n-1}$ ...(4.40)

Eqn. (4.39) is true for any working substance undergoing a reversible polytropic process. It follows also that for any polytropic process, we can write

$$\frac{p_2}{p_1} = \left(\frac{v_1}{v_2}\right)^n \quad \text{...(4.41)}$$

The following relations can be derived (following the same procedure as was done under reversible adiabatic process)

$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{n-1} \quad \text{...(4.42)}$$

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} \quad \text{...(4.43)}$$

Heat transfer during polytropic process (for perfect gas pv = RT) :

Using non-flow energy equation, the heat flow/transfer during the process can be found,

i.e., $Q = (u_2 - u_1) + W$

$$= c_v(T_2 - T_1) + \frac{R(T_1 - T_2)}{n-1}$$

i.e., $Q = \dfrac{R(T_1 - T_2)}{n-1} - c_v\,(T_1 - T_2)$

Also $c_v = \dfrac{R}{(\gamma - 1)}$

On substituting,

$$Q = \frac{R}{n-1}(T_1 - T_2) - \frac{R}{(\gamma-1)}\,(T_1 - T_2)$$

i.e., $Q = R(T_1 - T_2)\left(\dfrac{1}{n-1} - \dfrac{1}{\gamma-1}\right)$

$$= \frac{R\,(T_1 - T_2)(\gamma - 1 - n + 1)}{(\gamma-1)(n-1)} = \frac{R(T_1 - T_2)(\gamma - n)}{(\gamma-1)(n-1)}$$

∴ $Q = \dfrac{(\gamma - n)}{(\gamma - 1)}\dfrac{R(T_1 - T_2)}{(n-1)}$

or $Q = \left(\dfrac{\gamma - n}{\gamma - 1}\right)W \quad \left[\because W = \dfrac{R(T_1 - T_2)}{(n-1)}\right]$ ...(4.44)

In a polytropic process, the *index n depends only on the heat and work quantities* during the process. The various processes considered earlier are special cases of polytropic process for a perfect gas. For example,

When $n = 0$ pv^0 = constant *i.e.*, p = constant

When $n = \infty$ pv^∞ = constant

or $p^{1/\infty}\,v$ = constant, *i.e.*, v = constant

When $n = 1$ pv = constant, *i.e.*, T = constant

[since $(pv)/T$ = constant for a perfect gas]

When $n = \gamma$ pv^γ = constant, *i.e.*, reversible adiabatic

This is illustrated on a p-v diagram in Fig. 4.9.

(*i*) State 1 to state A is *constant pressure cooling* ($n = 0$).

(*ii*) State 1 to state B is *isothermal compression* ($n = 1$).

(*iii*) State 1 to state C is *reversible adiabatic compression* ($n = \gamma$).

(*iv*) State 1 to state D is *constant volume heating* ($n = \infty$).

Similarly,

(*i*) State 1 to state A' is *constant pressure heating* ($n = 0$).

(*ii*) State 1 to state B' is *isothermal expansion* ($n = 1$).

(*iii*) State 1 to state C' is *reversible adiabatic expansion* ($n = \gamma$).

(*iv*) State 1 to state D' is *constant volume cooling* ($n = \infty$).

It may be noted that, since γ is always greater than unity, than process 1 to C must lie between processes 1 to B and 1 to D ; similarly, process 1 to C' must lie between processes 1 to B' and 1 to D'.

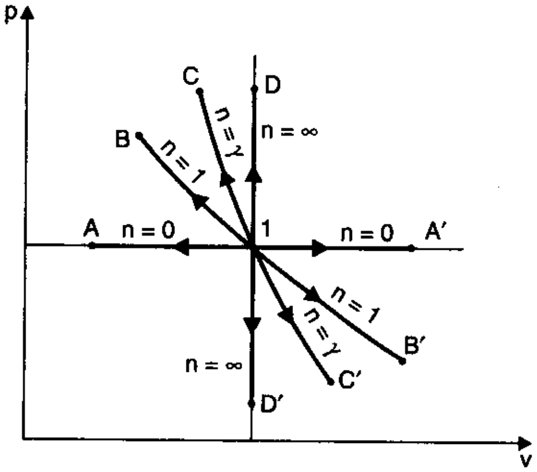


Fig. 4.9

6. Free Expansion

Consider two vessels 1 and 2 interconnected by a short pipe with a valve A, and perfectly thermally insulated [Fig. 4.10]. Initially let the vessel 1 be filled with a fluid at a certain pressure, and let 2 be completely evacuated. When the valve A is opened the fluid in 1 will expand rapidly to fill both vessels 1 and 2. The pressure finally will be lower than the initial pressure in vessel 1. This is known as *free or unresisted expansion*. The process is *highly irreversible* ; since the fluid is eddying continuously during the process. Now applying first law of thermodynamics (or non-flow energy equation) between the initial and final states,

$$Q = (u_2 - u_1) + W$$

In this process, no work is done on or by the fluid, since the boundary of the system does not move. No heat flows to or from the fluid since the system is well lagged. The process is therefore, *adiabatic but irreversible*.

i.e., $$u_2 - u_1 = 0 \quad \text{or} \quad u_2 = u_1$$

In a free expansion, therefore, the internal energy initially equals the initial energy finally.

For a perfect gas,

$$u = c_v T$$

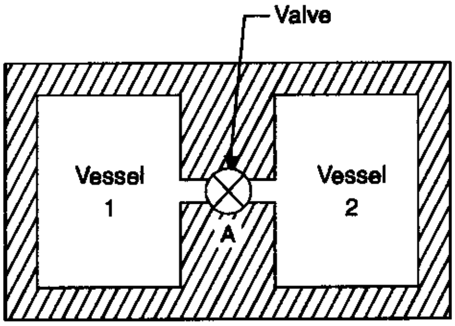


Fig. 4.10. Free expansion.

∴ For a free expansion of a perfect gas,

$$c_v T_1 = c_v T_2 \quad i.e., \quad T_1 = T_2$$

That is, for a perfect gas undergoing a free expansion, the initial temperature is equal to the final temperature.

Table 4.1
Summary of Processes for Perfect Gas (Unit mass)

Process	*Index* n	*Heat added*	$\int_1^2 pdv$	*p, v, T relations*	*Specific heat, c*
Constant pressure	$n = 0$	$c_p(T_2 - T_1)$	$p(v_2 - v_1)$	$\frac{T_2}{T_1} = \frac{v_2}{v_1}$	c_p
Constant volume	$n = \infty$	$c_v(T_2 - T_1)$	0	$\frac{T_1}{T_2} = \frac{p_1}{p_2}$	c_v
Constant temperature	$n = 1$	$p_1 v_1 \log_e \frac{v_2}{v_1}$	$p_1 v_1 \log_e \frac{v_2}{v_1}$	$p_1 v_1 = p_2 v_2$	∞
Reversible adiabatic	$n = \gamma$	0	$\frac{p_1 v_1 - p_2 v_2}{\gamma - 1}$	$p_1 v_1^{\gamma} = p_2 v_2^{\gamma}$ $\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{\gamma - 1}$ $= \left(\frac{p_2}{p_1}\right)^{\frac{\gamma - 1}{\gamma}}$	0
Polytropic	$n = n$	$c_n(T_2 - T_1)$ $= c_v \left(\frac{\gamma - n}{1 - n}\right)$ $\times (T_2 - T_1)$ $= \frac{\gamma - n}{\gamma - 1} \times$ work done (non-flow)	$\frac{p_1 v_1 - p_2 v_2}{n - 1}$	$p_1 v_1^{n} = p_2 v_2^{n}$ $\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{n - 1}$ $= \left(\frac{p_2}{p_1}\right)^{\frac{n - 1}{n}}$	$c_n = c_v \left(\frac{\gamma - n}{1 - n}\right)$

Note. Equations must be used keeping dimensional consistence.

Example 4.1. *In an internal combustion engine, during the compression stroke the heat rejected to the cooling water is 50 kJ/kg and the work input is 100 kJ/kg.*

Calculate the change in internal energy of the working fluid stating whether it is a gain or loss.

Solution. Heat rejected to the cooling water, $Q = -50$ kJ/kg

(–ve sign since heat is rejected)

Work input, $W = -100$ kJ/kg

(–ve sign since work is supplied to the system)

Using the relation, $Q = (u_2 - u_1) + W$

$$-50 = (u_2 - u_1) - 100$$

or $$u_2 - u_1 = -50 + 100 = 50 \text{ kJ/kg}$$

Hence, **gain in internal energy = 50 kJ/kg. (Ans.)**

Example 4.2. *In an air motor cylinder the compressed air has an internal energy of 450 kJ/kg at the beginning of the expansion and an internal energy of 220 kJ/kg after expansion. If the work done by the air during the expansion is 120 kJ/kg, calculate the heat flow to and from the cylinder.*

Solution. Internal energy at beginning of the expansion,

$$u_1 = 450 \text{ kJ/kg}$$

Internal energy after expansion,

$$u_2 = 220 \text{ kJ/kg}$$

Work done by the air during expansion,

$$W = 120 \text{ kJ/kg}$$

Heat flow, Q :

Using the relation, $Q = (u_2 - u_1) + W$

$\therefore$ $$Q = (220 - 450) + 120$$

$$= -230 + 120 = -110 \text{ kJ/kg}$$

Hence, **heat rejected by air = 110 kJ/kg. (Ans.)**

Example 4.3. *0.3 kg of nitrogen gas at 100 kPa and 40°C is contained in a cylinder. The piston is moved compressing nitrogen until the pressure becomes 1 MPa and temperature becomes 160°C. The work done during the process is 30 kJ.*

Calculate the heat transferred from the nitrogen to the surroundings.

c_v for nitrogen = 0.75 kJ/kg K.

Solution. Mass of nitrogen, $m = 0.3$ kg

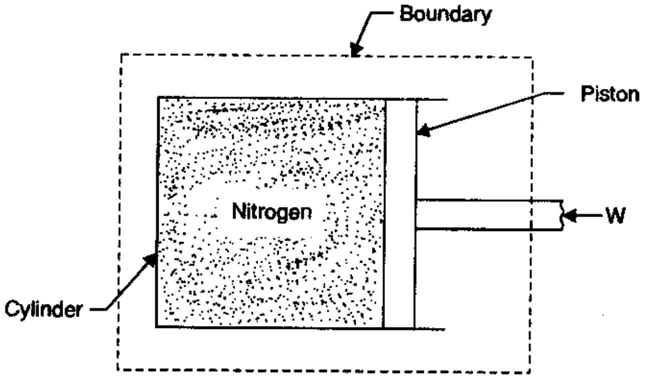


Fig. 4.11

Temperature before compression = 40°C or 313 K

Temperature after compression = 160°C or 433 K

The work done during the compression process, $W = -30$ kJ

According to first law of thermodynamics,

$$Q = \Delta U + W = (U_2 - U_1) + W$$
$$= mc_v\,(T_2 - T_1) + W$$
$$= 0.3 \times 0.75(433 - 313) - 30 = -3 \text{ kJ}$$

Hence, **heat 'rejected' during the process = 3 kJ. (Ans.)**

Note. Work, W has been taken –ve because it has been supplied from outside.

Example 4.4. *When a stationary mass of gas was compressed without friction at constant pressure its initial state of 0.4 m³ and 0.105 MPa was found to change to final state of 0.20 m³ and 0.105 MPa. There was a transfer of 42.5 kJ of heat from the gas during the process.*

How much did the internal energy of the gas change ?

Solution.

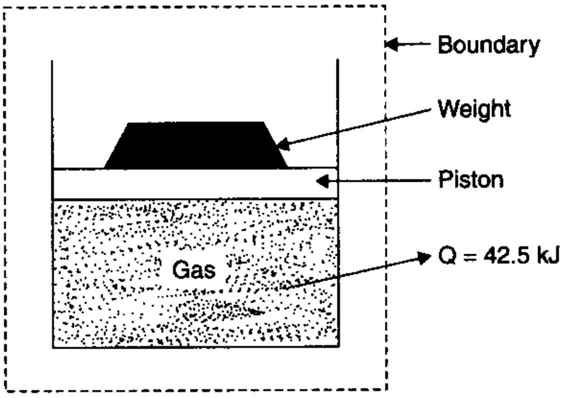


Fig. 4.12

Initial state

Pressure of gas, $p_1 = 0.105$ MPa

Volume of gas, $V_1 = 0.4 \text{ m}^3$

Final state

Pressure of gas, $p_2 = 0.105$ MPa

Volume of gas, $V_2 = 0.20 \text{ m}^3$

Process used : *Constant pressure*

Heat transferred, $Q = -42.5$ kJ

(–ve sign indicates that heat is rejected)

Change in internal energy, $\Delta U = U_2 - U_1$:

First law for a stationary system in a process gives

$$Q = \Delta U + W$$

or
$$Q_{1-2} = (U_2 - U_1) + W_{1-2} \quad ...(i)$$

Here
$$W_{1-2} = \int_{V_1}^{V_2} pdV = p(V_2 - V_1)$$

$$= 0.105(0.20 - 0.40) \text{ MJ} = -21 \text{ kJ} \qquad [\because \ 1 \text{ MJ} = 10^3 \text{ kJ}]$$

Substituting this value of W_{1-2} in equation (*i*), we get

$$-42.5 = (U_2 - U_1) - 21$$

$$\therefore \quad U_2 - U_1 = -42.5 + 21 = -21.5 \text{ kJ}$$

Hence **'decrease' in internal energy = 21.5 kJ. (Ans.)**

Example 4.5. *A container is divided into compartments by a partition. The container is completely insulated so that there is no heat transfer. One portion contains gas at temperature* T_1 *and pressure* p_1 *while the other portion also has the same gas but at temperature* T_2 *and pressure* p_2.

How will the First Law of Thermodynamics conclude the result if partition is removed ?

Solution. Refer Fig. 4.13.

According to First Law of Thermodynamics,

$$\delta Q = \delta U + \delta W$$

When partition removed, $\delta Q = 0$

$$\delta W = 0$$

$$\therefore \quad \delta U = 0.$$

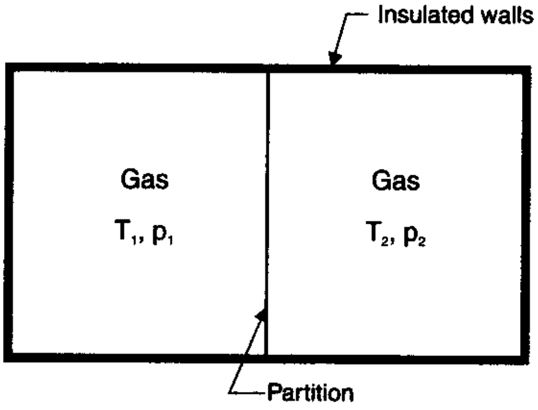


Fig. 4.13

Conclusion. *There is conservation of internal energy.*

Example 4.6. *Air enters a compressor at* 10^5 *Pa and 25°C having volume of 1.8* m^3/kg *and is compressed to 5 ×* 10^5 *Pa isothermally.*

Determine : (i) Work done ;

(ii) Change in internal energy ; and

(iii) Heat transferred.

Solution. Initial pressure of air, $p_1 = 10^5$ Pa

Initial temperature of air, $T_1 = 25 + 273 = 298$ K

Final pressure of air, $p_2 = 5 \times 10^5$ Pa

Final temperature of air, $T_2 = T_1 = 298$ K (isothermal process)

Since, it is a closed steady state process, we can write down the first law of thermodynamics as,

$$Q = (u_2 - u_1) + W \text{ ......per kg}$$

(*i*) *For isothermal process :*

$$W_{1-2} = \int_1^2 p \, . \, dv = p_1 v_1 \log_e \left(\frac{p_1}{p_2}\right)$$

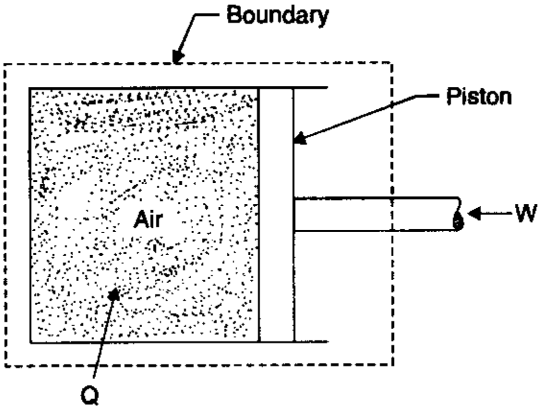


Fig. 4.14

as $p_1 v_1 = p_2 v_2$ for isothermal process

$$\therefore \quad W_{1-2} = -10^5 \times 1.8 \log_e \left(\frac{1 \times 10^5}{5 \times 10^5}\right)$$

$$= -2.897 \times 10^5 = -289.7 \text{ kJ/kg.}$$

(– ve sign indicates that the work is supplied to the air)

∴ **Work done on the air = 289.7 kJ/kg. (Ans.)**

(*ii*) Since temperature is constant,

$$\therefore \quad u_2 - u_1 = 0$$

∴ **Change in internal energy = zero. (Ans.)**

(*iii*) Again, $Q_{1-2} = (u_2 - u_1) + W$

$$= 0 + (-289.7) = -289.7 \text{ kJ}$$

(– ve sign indicates that heat is lost from the system to the surroundings)

∴ **Heat rejected = 289.7 kJ/kg. (Ans.)**

Example 4.7. *A cylinder containing the air comprises the system. Cycle is completed as follows :*

(*i*) *82000 N-m of work is done by the piston on the air during compression stroke and 45 kJ of heat are rejected to the surroundings.*

(*ii*) *During expansion stroke 100000 N-m of work is done by the air on the piston.*

Calculate the quantity of heat added to the system.

Solution. Refer Fig. 4.15.

Compression stroke. Process 1-2 :

Work done by the piston *on the air,* $W_{1-2} = -82000$ N-m (= – 82 kJ)

Heat rejected to the system, $Q_{1-2} = -45$ kJ

Now, $Q_{1-2} = (U_2 - U_1) + W$

$$-45 = (U_2 - U_1) + (-82)$$

$$\therefore \quad (U_2 - U_1) = 37 \text{ kJ} \qquad ...(i)$$

Expansion stroke. Process 2-1 :

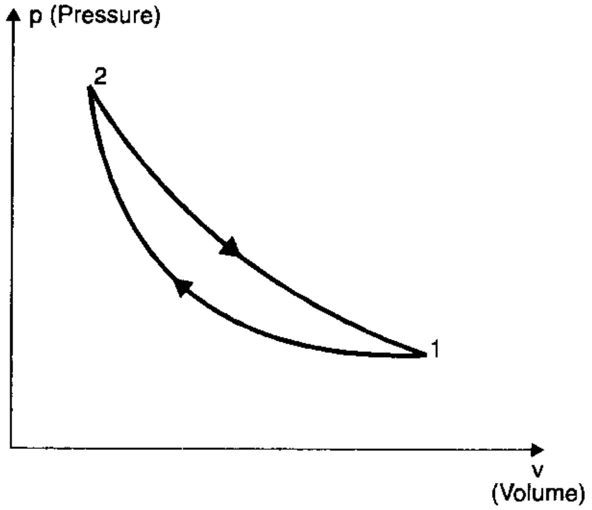


Fig. 4.15

Work done by air on the piston, W_{2-1} = 100000 N-m (= 100 kJ)

Now,
$$Q_{2-1} = (U_1 - U_2) + W$$
$$= -37 + 100 \text{ kJ} = 63 \text{ kJ}$$

Hence, **quantity of heat added to the system = 63 kJ. (Ans.)**

☞**Example 4.8.** *A tank containing air is stirred by a paddle wheel. The work input to the paddle wheel is 9000 kJ and the heat transferred to the surroundings from the tank is 3000 kJ.*

Determine : (i) Work done ;

(ii) Change in internal energy of the system.

Solution. Refer Fig. 4.16.

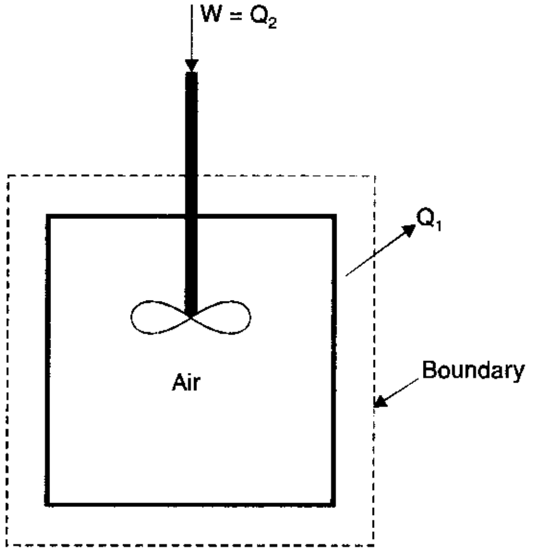


Fig. 4.16

Work input to the paddle wheel = 9000 kJ

Heat transferred to the surroundings from the tank = 3000 kJ

As it is a closed system, the first law of thermodynamics can be written as

$$U_1 - Q + W = U_2 \qquad \text{...}(i)$$

The *work* enters into the tank in the form of energy only so this should be considered as *heat input*.

$$\therefore \qquad Q = Q_1 - Q_2$$
$$= 3000 - 9000 = -\,6000 \text{ kJ.}$$

(*i*) Since volume does not change (being constant volume process)

$\therefore$ Work done, $\qquad W = 0$

Putting the value of $W = 0$ in equation (*i*), we get

(*ii*) $\qquad U_1 - (-\,6000) + 0 = U_2$

$\therefore \qquad U_2 - U_1 = 6000$ kJ

Hence, **change in internal energy (increase) = 6000 kJ. (Ans.)**

Example 4.9. *A stone of 20 kg mass and a tank containing 200 kg water comprise a system. The stone is 15 m above the water level initially. The stone and water are at the same temperature initially. If the stone falls into water, then determine* ΔU, ΔPE, ΔKE, Q *and* W, *when*

(*i*) *the stone is about to enter the water,*

(*ii*) *the stone has come to rest in the tank, and*

(*iii*) *the heat is transferred to the surroundings in such an amount that the stone and water come to their initial temperature.*

Solution. Refer Fig. 4.17.

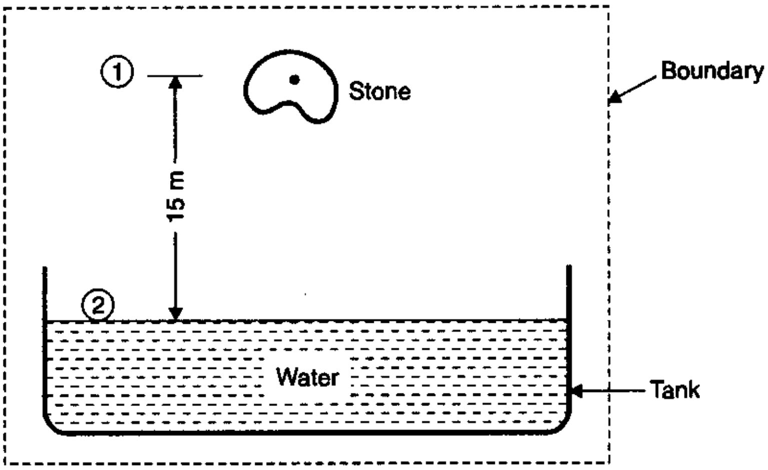


Fig. 4.17

Mass of stone = 20 kg

Mass of water in the tank = 200 kg

Height of stone above water level = 15 m

Applying the first law of thermodynamics,

$$Q = (U_2 - U_1) + m\left[\frac{C_2^2 - C_1^2}{2}\right] + mg\,(Z_2 - Z_1) + W$$
$$= \Delta U + \Delta KE + \Delta PE + W \qquad \text{...}(1)$$

Here Q = Heat leaving the boundary.

(*i*) When the stone is about to enter the water,

$$Q = 0,\ W = 0,\ \Delta U = 0$$

$$\because \quad -\Delta KE = \Delta PE = mg\,(Z_2 - Z_1)$$

$$= 20 \times 9.81\,(0 - 15) = -2943 \text{ J}$$

$$\therefore \quad \Delta KE = 2943 \text{ J}$$

and $\Delta PE = \mathbf{-2943\ J.}$ **(Ans.)**

(*ii*) When the stone dips into the tank and comes to rest

$$Q = 0,\ W = 0,\ \Delta KE = 0$$

Substituting these values in eqn. (1), we get

$$0 = \Delta U + 0 + \Delta PE + 0$$

$$\therefore \quad \Delta U = -\Delta PE = -(-2943) = \mathbf{2943\ J.}$$ **(Ans.)**

This shows that the internal energy (temperature) of the system increases.

(*iii*) When the water and stone come to their initial temperature,

$$W = 0,\ \Delta KE = 0$$

Substituting these values in eqn. (1), we get

$$\therefore \quad Q = -\Delta U = \mathbf{-2943\ J.}$$ **(Ans.)**

The *negative* sign shows that the *heat is lost from the system to the surroundings.*

☞**Example 4.10.** *When a system is taken from state l to state m, in Fig. 4.18, along path lqm, 168 kJ of heat flows into the system, and the system does 64 kJ of work :*

(i) How much will be the heat that flows into the system along path lnm if the work done is 21 kJ ?

(ii) When the system is returned from m to l along the curved path, the work done on the system is 42 kJ. Does the system absorb or liberate heat, and how much of the heat is absorbed or liberated ?

(iii) If $U_l = 0$ *and* $U_n = 84$ *kJ, find the heat absorbed in the processes ln and nm.*

Solution. Refer Fig. 4.18.

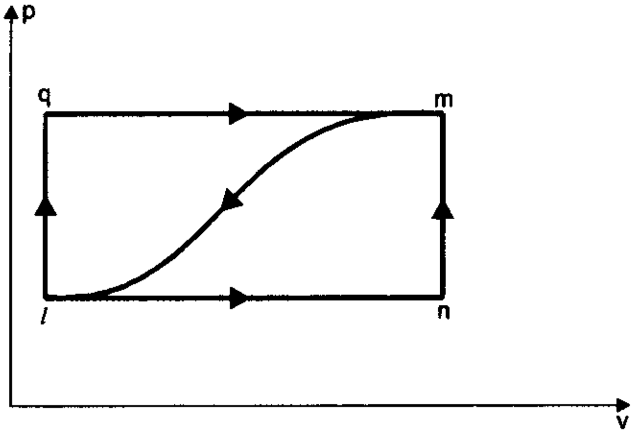


Fig. 4.18

$$Q_{l-q-m} = 168 \text{ kJ}$$

$$W_{l-q-m} = 64 \text{ kJ}$$

We have, $Q_{l-q-m} = (U_m - U_l) + W_{l-q-m}$

$168 = (U_m - U_l) + 64$

$\therefore$ $U_m - U_l =$ **104 kJ. (Ans.)**

(*i*) $Q_{l-n-m} = (U_m - U_l) + W_{l-n-m}$

$= 104 + 21 =$ **125 kJ. (Ans.)**

(*ii*) $Q_{m-l} = (U_l - U_m) + W_{m-l}$

$= -104 + (-42) =$ **– 146 kJ. (Ans.)**

The system liberates 146 kJ.

(*iii*) $W_{l-n-m} = W_{l-n} + W_{n-m} = W_{l-m} = 21$ kJ

[$\because$ $W_{n-m} = 0$, since volume does not change.]

$\therefore$ $Q_{l-n} = (U_n - U_l) + W_{l-n}$

$= (84 - 0) + 21 =$ **105 kJ. (Ans.)**

Now $Q_{l-m-n} = 125 \text{ kJ} = Q_{l-n} + Q_{n-m}$

$\therefore$ $Q_{n-m} = 125 - Q_{l-n}$

$= 125 - 105 =$ **20 kJ. (Ans.)**

Example 4.11. *In a system, executing a non-flow process, the work and heat per degree change of temperature are given by*

$$\frac{dW}{dT} = 200 \text{ W-s/°C and } \frac{dQ}{dT} = 160 \text{ J/°C}$$

What will be the change of internal energy of the system when its temperature changes from

$$T_1 = 55°C \text{ to } T_2 = 95°C \text{ ?}$$

Solution. Initial temperature, $T_1 = 55°C$; Final temperature, $T_2 = 95°C$

$$\frac{dW}{dT} = 200 \text{ W-s/°C} \ ; \ \frac{dQ}{dT} = 160 \text{ J/°C}.$$

Change of internal energy :

Now, $\frac{dW}{dT} = 200$ W-s/°C

$$\therefore \quad W = \int_{T_1}^{T_2} 200\, dT = \Big| 200\, T \Big|_{T_1}^{T_2} = \Big| 200\, T \Big|_{55}^{95}$$

$= 200\,(95 - 55) = 8000$ W-s $= 8000$ J [$\because$ 1 W-s = 1 J]

Also, $\frac{dQ}{dT} = 160$ J/°C

$$\therefore \quad Q = \int_{T_1}^{T_2} 160\, dT = \Big| 160\, T \Big|_{T_1}^{T_2}$$

$$= \Big| 160\, T \Big|_{55}^{95} = 160\,(95 - 55) = 6400 \text{ J}$$

Applying the first law of thermodynamics to the given non-flow system,

$$Q = \Delta U + W$$

$$\therefore \qquad 6400 = \Delta U + 8000$$

or
$$\Delta U = -1600 \text{ J} = \mathbf{-1.6 \text{ kJ}}. \quad \textbf{(Ans.)}$$

The –ve sign indicates that there is *decrease* in internal energy.

Example 4.12. *A fluid system, contained in a piston and cylinder machine, passes through a complete cycle of four processes. The sum of all heat transferred during a cycle is – 340 kJ. The system completes 200 cycles per min.*

Complete the following table showing the method for each item, and compute the net rate of work output in kW.

Process	*Q (kJ/min)*	*W (kJ/min)*	*ΔE (kJ/min)*
1—2	*0*	*4340*	—
2—3	*42000*	*0*	—
3—4	*– 4200*	—	*– 73200*
4—1	—	—	—

Solution. Sum of all heat transferred during the cycle = – 340 kJ.

Number of cycles completed by the system = 200 cycles/min.

Process 1—2 :

$$Q = \Delta E + W$$

$$0 = \Delta E + 4340$$

$$\therefore \qquad \Delta E = -4340 \text{ kJ/min.}$$

Process 2—3 :

$$Q = \Delta E + W$$

$$42000 = \Delta E + 0$$

$$\Delta E = 42000 \text{ kJ/min.}$$

Process 3—4 :

$$Q = \Delta E + W$$

$$-4200 = -73200 + W$$

$$\therefore \qquad W = 69000 \text{ kJ/min.}$$

Process 4—1 :

$$\underset{\text{cycle}}{\Sigma Q} = -340 \text{ kJ}$$

The system completes 200 cycles/min

$$\because \quad Q_{1-2} = Q_{2-3} + Q_{3-4} + Q_{4-1} = -340 \times 200 = -68000 \text{ kJ/min}$$

or
$$0 + 42000 + (-4200) + Q_{4-1} = -68000$$

$$Q_{4-1} = -105800 \text{ kJ/min.}$$

Now, $\int dE = 0$, since cyclic integral of any property is zero.

$$\Delta E_{1-2} + \Delta E_{2-3} + \Delta E_{3-4} + \Delta E_{4-1} = 0$$

$$-4340 + 42000 + (-73200) + \Delta E_{4-1} = 0$$

$$\therefore \qquad \Delta E_{4-1} = 35540 \text{ kJ/min.}$$

$$\therefore \qquad W_{4-1} = Q_{4-1} - \Delta E_{4-1}$$

$$= -105800 - 35540 = -141340 \text{ kJ/min}$$

The completed table is given below :

Process	*Q(kJ/min)*	*W(kJ/min)*	ΔE*(kJ/min)*
1—2	0	4340	– 4340
2—3	42000	0	42000
3—4	– 4200	69000	– 73200
4—1	– 105800	– 141340	35540

Since $$\sum_{\text{cycle}} Q = \sum_{\text{cycle}} W$$

Rate of work output = – 68000 kJ/min $= -\dfrac{68000}{60}$ kJ/s or kW

= **1133.33 kW. (Ans.)**

Example 4.13. *The power developed by a turbine in a certain steam plant is 1200 kW. The heat supplied to the steam in the boiler is 3360 kJ/kg, the heat rejected by the system to cooling water in the condenser is 2520 kJ/kg and the feed pump work required to pump the condensate back into the boiler is 6 kW.*

Calculate the steam flow round the cycle in kg/s.

Solution. The power developed by the turbine = 1200 kW

The heat supplied to the steam in the boiler = 3360 kJ/kg

The heat rejected by the system to cooling water = 2520 kJ/kg

Feed pump work = 6 kW

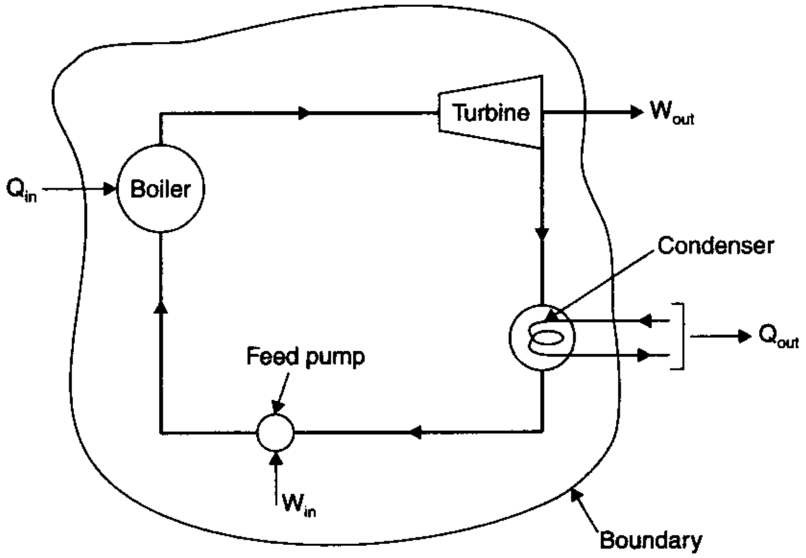


Fig. 4.19

Fig. 4.19 shows the cycle. A boundary is shown which encompasses the entire plant. Strictly, this boundary should be thought of as encompassing the working fluid only.

$$\oint dQ = 3360 - 2520 = 840 \text{ kJ/kg}$$

Let the system flow be in kg/s.

$$\therefore \quad \oint dQ = 840\ \dot{m}\ \text{kJ/s}$$

$$\oint dW = 1200 - 6 = 1194\ \text{kJ/s}$$

But $$\oint dQ = \oint dW$$

i.e., $$840\ \dot{m} = 1194$$

$$\therefore \quad \dot{m} = \frac{1194}{840} = 1.421\ \text{kg/s}$$

$\because$ **Steam flow round the cycle = 1.421 kg/s. (Ans.)**

Example 4.14. *A closed system of constant volume experiences a temperature rise of 25°C when a certain process occurs. The heat transferred in the process is 30 kJ. The specific heat at constant volume for the pure substance comprising the system is 1.2 kJ/kg°C, and the system contains 2.5 kg of this substance. Determine :*

(i) The change in internal energy ;

(ii) The work done.

Solution. Temperature rise, $(T_2 - T_1) = 25°\text{C}$

The heat transferred in the process, $Q = 30$ kJ

Specific heat at constant volume, $c_v = 1.2$ kJ/kg°C

Mass of the substance, $m = 2.5$ kg

Now, $$\Delta U = m \int_{T_1}^{T_2} c_v\, dT$$

$$= 2.5 \int_{T_1}^{T_2} 1.2\, dT = 3.0 \times (T_2 - T_1)$$

$$= 3.0 \times 25 = \mathbf{75\ kJ}$$

Hence, **the change in internal energy is 75 kJ. (Ans.)**

According to the first law of thermodynamics,

$$Q = \Delta U + W$$

$$\therefore \quad 30 = 75 + W$$

$$\therefore \quad W = 30 - 75 = -45\ \text{kJ}$$

Hence, **the work done = – 45 kJ. (Ans.)**

It may be observed that *even though the volume is constant the work is not zero.* Clearly, the process is **irreversible.**

Example 4.15. *A system receives 50 kJ of heat while expanding with volume change of 0.14 m^3 against an atmosphere of 1.2×10^5 N/m^2. A mass of 90 kg in the surroundings is also lifted through a distance of 5.5 metres.*

(i) Find the change in energy of the system.

(ii) The system is returned to its initial volume by an adiabatic process which requires 110 kJ of work. Find the change in energy of the system.

(iii) For the combined processes of (i) and (ii) determine the change in energy of the system.

Solution. Heat received by the system,

$$Q = 50 \text{ kJ}$$

Change in volume $\Delta V = 0.14 \text{ m}^3$

Pressure $= 1.2 \times 10^5 \text{ N/m}^2$

Mass lifted in the surroundings $= 90$ kg

Distance through which lifted $= 5.5$ m

Work done during adiabatic process $= -110$ kJ.

(*i*) $$Q = \Delta E + W \qquad ...(i)$$

Now, $$W = p.\Delta V + W_{net}$$

$$= \left(\frac{1.2 \times 10^5 \times 0.14}{1000}\right) + \left(\frac{90 \times 5.5 \times 9.8}{1000}\right) \text{kJ}$$

$$= 16.8 + 4.85 = 21.65 \text{ kJ}$$

But [from (*i*)], $$\Delta E = Q - W$$

$$= 50 - 21.65 = \mathbf{28.35 \text{ kJ.}} \quad \textbf{(Ans.)}$$

(*ii*) Since the process is adiabatic,

$$Q = 0$$

and $$\Delta E = -W$$

$$= -(-110) = \mathbf{110 \text{ kJ.}} \quad \textbf{(Ans.)}$$

(*iii*) Change in internal energy,

$$\Delta E = Q - W$$

$$= 50 - [(-110) + 21.65] = \mathbf{138.35 \text{ kJ.}} \quad \textbf{(Ans.)}$$

☞**Example 4.16.** *A fluid system undergoes a non-flow frictionless process following the pressure-volume relation as* $p = \frac{5}{V} + 1.5$ *where p is in bar and V is in* m^3. *During the process the volume changes from 0.15* m^3 *to 0.05* m^3 *and the system rejects 45 kJ of heat. Determine :*

(*i*) *Change in internal energy ;*

(*ii*) *Change in enthalpy.*

Solution. Pressure-volume relation : $p = \frac{5}{V} + 1.5$

Initial volume, $V_1 = 0.15 \text{ m}^3$

Final volume, $V_2 = 0.05 \text{ m}^3$

Heat *rejected* by the system, $Q = -45$ kJ

Work done is given by,

$$W = \int_1^2 p \,.\, dV = \int_{V_1}^{V_2} \left(\frac{5}{V} + 1.5\right) dV$$

$$= \int_{0.15}^{0.05} \left(\frac{5}{V} + 1.5\right) dV = \left[5 \log_e \frac{V_2}{V_1} + 1.5\,(V_2 - V_1)\right]_{0.15}^{0.05} \times 10^5 \text{ N-m}$$

$$= 10^5 \left[5 \log_e \frac{0.05}{0.15} + 1.5\,(0.05 - 0.15)\right] = 10^5(-5.49 - 0.15) \text{ N-m}$$

$$= -5.64 \times 10^5 \text{ N-m} = -5.64 \times 10^5 \text{ J} \qquad [\because \; 1 \text{ Nm} = 1 \text{ J}]$$

$$= -564 \text{ kJ.}$$

(*i*) Applying the first law energy equation,

$$Q = \Delta U + W$$

$$-45 = \Delta U + (-564)$$

$$\therefore \quad \mathbf{\Delta U = 519\ kJ.}\quad \textbf{(Ans.)}$$

This shows that the internal energy is **increased.**

(*ii*) Change in enthalpy,

$$\Delta H = \Delta U + \Delta (pV)$$

$$= 519 \times 10^3 + (p_2V_2 - p_1V_1)$$

$$p_1 = \frac{5}{V_1} + 1.5 = \frac{5}{0.15} + 1.5 = 34.83 \text{ bar}$$

$$= 34.83 \times 10^5 \text{ N/m}^2$$

$$p_2 = \frac{5}{V_2} + 1.5 = \frac{5}{0.05} + 1.5$$

$$= 101.5 \text{ bar} = 101.5 \times 10^5 \text{ N/m}^2$$

$$\therefore \quad \Delta H = 519 \times 10^3 + (101.5 \times 10^5 \times 0.05 - 34.83 \times 10^5 \times 0.15)$$

$$= 519 \times 10^3 + 10^3(507.5 - 522.45)$$

$$= 10^3(519 + 507.5 - 522.45) = 504 \text{ kJ}$$

∴ **Change in enthalpy = 504 kJ. (Ans.)**

☞**Example 4.17.** *The following equation gives the internal energy of a certain substance*

$$u = 3.64\, pv + 90$$

where u is kJ/kg, p is in kPa and v is in m^3*/kg.*

A system composed of 3.5 kg of this substance expands from an initial pressure of 500 kPa and a volume of 0.25 m^3 *to a final pressure 100 kPa in a process in which pressure and volume are related by* $pv^{1.25}$ *= constant.*

(*i*) *If the expansion is quasi-static, find Q,* ΔU *and W for the process.*

(*ii*) *In another process, the same system expands according to the same pressure-volume relationship as in part (i), and from the same initial state to the same final state as in part (i), but the heat transfer in this case is 32 kJ. Find the work transfer for this process.*

(*iii*) *Explain the difference in work transfer in parts (i) and (ii).*

Solution. Internal energy equation : $u = 3.64\, pv + 90$

Initial volume, $V_1 = 0.25 \text{ m}^3$

Initial pressure, $p_1 = 500$ kPa

Final pressure, $p_2 = 100$ kPa

Process : $pv^{1.25}$ = constant.

(*i*) Now, $u = 3.64\, pv + 90$

$$\Delta u = u_2 - u_1$$

$$= 3.64\,(p_2v_2 - p_1v_1) \qquad \text{...per kg}$$

$$\therefore \quad \Delta U = 3.64\,(p_2V_2 - p_1V_1) \qquad \text{...for 3.5 kg}$$

Now, $p_1V_1^{1.25} = p_2V_2^{1.25}$

$$V_2 = V_1\left(\frac{p_1}{p_2}\right)^{1/1.25} = 0.25\left(\frac{500}{100}\right)^{1/1.25}$$

$$= 0.906 \text{ m}^3$$

$$\therefore \quad \Delta U = 3.64\,(100 \times 10^3 \times 0.906 - 500 \times 10^3 \times 0.25)\text{ J} \quad [\because \ 1 \text{ Pa} = 1 \text{ N/m}^2]$$
$$= 3.64 \times 10^5\,(0.906 - 5 \times 0.25)\text{ J}$$
$$= -\,3.64 \times 10^5 \times 0.344 \text{ J} = -\,125.2 \text{ kJ}$$

i.e., $\mathbf{\Delta U = -\,125.2 \ kJ.}$ **(Ans.)**

For a **quasi-static process**

$$W = \int p dV = \frac{p_1V_1 - p_2V_2}{n-1}$$
$$= \frac{(500 \times 10^3 \times 0.25 - 100 \times 10^3 \times 0.906)}{(1.25-1)} = \frac{125 - 90.6}{0.25} \text{ kJ} = 137.6 \text{ kJ}$$
$$\therefore \quad Q = \Delta U + W$$
$$= -\,125.2 + 137.6 = 12.4 \text{ kJ}$$

i.e., $\mathbf{Q = 12.4 \ kJ.}$ **(Ans.)**

(*ii*) Here $Q = 32$ kJ

Since the end states are the same, ΔU would remain the same as in (*i*)

$$\therefore \quad W = Q - \Delta U = 32 - (-\,125.2)$$
$$= \mathbf{157.2 \ kJ.} \ \textbf{(Ans.)}$$

(*iii*) The work in (*ii*) is **not equal** to $\int p\, dV$ since the process is **not quasi-static.**

☞**Example 4.18.** *The properties of a system, during a reversible constant pressure non-flow process at p = 1.6 bar, changed from* $v_1 = 0.3$ *m³/kg,* $T_1 = 20°C$ *to* $v_2 = 0.55$ *m³/kg,* $T_2 = 260°C$*. The specific heat of the fluid is given by*

$$c_p = \left(1.5 + \frac{75}{T + 45}\right) \text{kJ/kg°C, where } T \text{ is in °C.}$$

Determine : (i) Heat added/kg ; (ii) Work done/kg ;
(iii) Change in internal energy/kg ; (iv) Change in enthalpy/kg.

Solution. Initial volume, $v_1 = 0.3$ m³/kg

Initial temperature, $T_1 = 20$°C

Final volume, $v_2 = 0.55$ m³/kg

Final temperature, $T_2 = 260$°C

Constant pressure, $p = 1.6$ bar

Specific heat at constant pressure, $c_p = \left(1.5 + \frac{75}{T + 45}\right)$ kJ/kg°C

(*i*) The **heat added** per kg of fluid is given by

$$Q = \int_{T_1}^{T_2} c_p \, dT = \int_{20}^{260} \left(1.5 + \frac{75}{T+45}\right) dT$$
$$= \Big| 1.5\,T + 75 \log_e (T + 45) \Big|_{20}^{260}$$
$$= 1.5\,(260 - 20) + 75 \times \log_e \left(\frac{260+45}{20+45}\right) = 475.94 \text{ kJ}$$

$\therefore$ **Heat added** $= \mathbf{475.94 \ kJ/kg.}$ **(Ans.)**

(*ii*) The *work done* per kg of fluid is given by

$$W = \int_{v_1}^{v_2} p\,dv = p(v_2 - v_1) = 1.6 \times 10^5 (0.55 - 0.3) \text{ N-m}$$

$$= 40 \times 10^3 \text{ J} = 40 \text{ kJ}$$

∴ **Work done = 40 kJ/kg. (Ans.)**

(*iii*) **Change in internal energy,**

$$\Delta u = Q - W = 475.94 - 40 = \mathbf{435.94 \text{ kJ/kg.}} \quad \textbf{(Ans.)}$$

(*iv*) **Change in enthalpy,** (for **non-flow process**)

$$\Delta h = Q = \mathbf{475.94 \text{ kJ/kg.}} \quad \textbf{(Ans.)}$$

Example 4.19. *1 kg of gaseous CO_2 contained in a closed system undergoes a reversible process at constant pressure. During this process 42 kJ of internal energy is decreased. Determine the work done during the process.*

Take $c_p = 840$ J/kg°C and $c_v = 600$ J/kg°C.

Solution. Mass CO_2, $m = 1$ kg

Decrease in internal energy, $\Delta u = -42 \text{ kJ} = -42 \times 10^3 \text{ J}$

Specific heat at constant pressure, $c_p = 840$ J/kg°C

Specific heat at constant volume, $c_v = 600$ J/kg°C

Let, initial temperature of CO_2 $= T_1$

Final temperature of CO_2 $= T_2$

Now change in internal energy,

$$\Delta U = m \times c_v (T_2 - T_1)$$

$$-42 \times 10^3 = 1 \times 600 (T_2 - T_1)$$

$$\therefore \quad T_2 - T_1 = -\frac{42 \times 10^3}{600} = -70°\text{C}$$

The heat supplied or rejected,

$$Q = mc_p (T_2 - T_1)$$

$$= 1 \times 840 \times (-70) = -58800 \text{ J or } -58.8 \text{ kJ}$$

Applying first law to the process,

$$Q = \Delta U + W$$

$$-58.8 = -42 + W \text{ or } W = -16.8 \text{ kJ}$$

∴ **Work done during the process = – 16.8 kJ. (Ans.)**

☞**Example 4.20.** *A fluid is contained in a cylinder by a spring-loaded, frictionless piston so that the pressure in the fluid is a linear function of the volume ($p = a + bV$). The internal energy of the fluid is given by the following equation*

$$U = 42 + 3.6\,pV$$

where U is in kJ, p in kPa, and V in cubic metre. If the fluid changes from an initial state of 190 kPa, 0.035 m³ to a final state of 420 kPa, 0.07 m³, with no work other than that done on the piston, find the direction and magnitude of the work and heat transfer.

Solution. Relation between pressure and volume, $p = a + bV$.

Equation of internal energy : $U = 42 + 3.6pV$

Initial pressure, $p_1 = 190$ kPa

Initial volume, $V_1 = 0.035$ m³

Final pressure, $p_2 = 420$ kPa

Final volume, $V_2 = 0.07$ m³

The change in internal energy of the fluid during the process

$$\begin{aligned} U_2 - U_1 &= (42 + 3.6p_2V_2) - (42 + 3.6p_1V_1) \\ &= 3.6(p_2V_2 - p_1V_1) \\ &= 3.6(4.2 \times 10^5 \times 0.07 - 1.9 \times 10^5 \times 0.035) \text{ J} \\ &= 360(4.2 \times 0.07 - 1.9 \times 0.035) \text{ kJ} \\ &= 81.9 \text{ kJ} \end{aligned}$$

Now, $p = a + bV$

$$190 = a + b \times 0.035 \qquad \text{...}(i)$$

$$420 = a + b \times 0.07 \qquad \text{...}(ii)$$

Subtracting (*i*) from (*ii*), we get

$$230 = 0.035\ b \text{ or } b = \frac{230}{0.035} = 6571 \text{ kN/m}^5$$

and $a = -\ 40 \text{ kN/m}^2$

Work transfer involved during the process

$$W_{1-2} = \int_{V_1}^{V_2} pdV = \int_{V_1}^{V_2} (a + bV)\,dV = a(V_2 - V_1) + b\left(\frac{V_2^2 - V_1^2}{2}\right)$$

$$= (V_2 - V_1)\left[a + \frac{b}{2}(V_1 + V_2)\right]$$

$$= (0.07 - 0.035)\left[-40 \text{ kN/m}^2 + \frac{6571}{2} \text{ kN/m}^5 (0.035 + 0.07)\right] = 10.67 \text{ kJ}$$

∴ **Work done by the system = 10.67 kJ. (Ans.)**

Heat transfer involved,

$$Q_{1-2} = (U_2 - U_1) + W_{1-2} = 81.9 + 10.67 = 92.57 \text{ kJ.}$$

92.57 kJ of heat flow into the system during the process. (Ans.)

Example 4.21. *90 kJ of heat are supplied to a system at a constant volume. The system rejects 95 kJ of heat at constant pressure and 18 kJ of work is done on it. The system is brought to original state by adiabatic process. Determine :*

(*i*) *The adiabatic work ;*

(*ii*) *The values of internal energy at all end states if initial value is 105 kJ.*

Solution. Refer Fig. 4.20.

Heat supplied at constant volume = 90 kJ

Heat rejected at constant pressure = – 95 kJ

Work done *on* the system = – 18 kJ

Initial value of internal energy, $U_l = 105$ kJ

Process *l–m* (*constant volume*) :

$$W_{l-m} = 0$$

$$Q_{l-m} = 90 = U_m - U_l$$

$$\therefore \quad U_m = U_l + 90 = 105 + 90$$

$$= 195 \text{ kJ}$$

Process *m–n* (*constant pressure*) :

$$Q_{m-n} = (U_n - U_m) + W_{m-n}$$

$$-\ 95 = (U_n - U_m) - 18$$

$$\therefore \quad U_n - U_m = -\ 77 \text{ kJ}$$

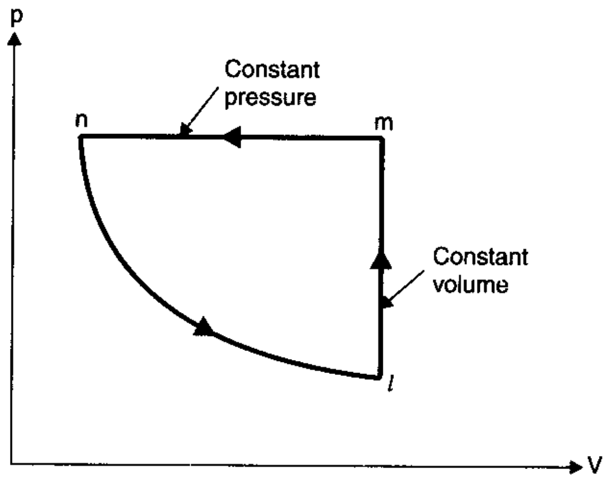


Fig. 4.20

$$\therefore \qquad U_n = 195 - 77 = 118 \text{ kJ}$$

$Q_{n-l} = 0$ being adiabatic process

$$\therefore \qquad \oint \delta Q = 90 - 95 = -5 \text{ kJ}$$

and

$$\oint \delta W = -18 + W_{n-l} = -5$$

$$\therefore \qquad W_{n-l} = -5 + 18 = 13 \text{ kJ}$$

Hence, $\mathbf{W_{n-l} = 13\ kJ\ ;\ U_l = 105\ kJ\ ;\ U_m = 195\ kJ\ ;\ U_n = 118\ kJ.}$ **(Ans.)**

Example 4.22. *A movable frictionless piston closes a fully insulated cylinder on one side and offers a constant resistance during its motion. A paddle work is drawn into the cylinder and does work on the system.*

Prove that the paddle work is equal to change in enthalpy.

Solution. Refer Fig. 4.21.

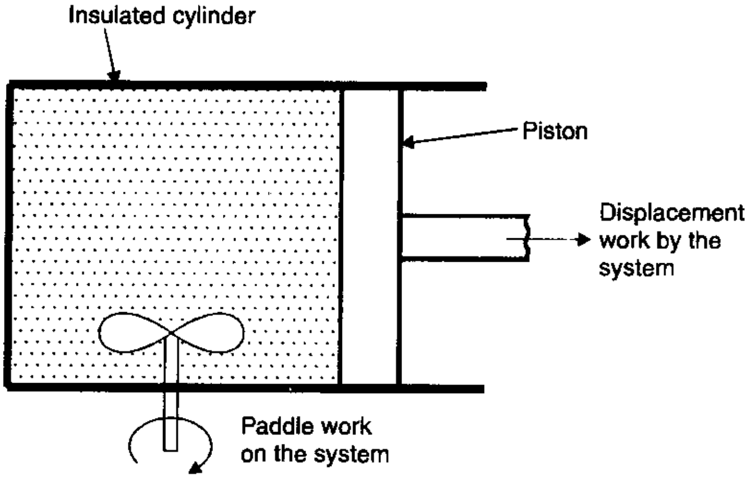


Fig. 4.21

$$Q = W_{paddle} = \Delta U + p\Delta V = \Delta U + \Delta(pV) = \Delta(U + pV) = \Delta H$$

Hence **paddle work is equal to change in enthalpy. (Ans.)**

☞**Example 4.23.** *0.2 m³ of air at 4 bar and 130°C is contained in a system. A reversible adiabatic expansion takes place till the pressure falls to 1.02 bar. The gas is then heated at constant pressure till enthalpy increases by 72.5 kJ. Calculate :*

(i) The work done ;

(ii) The index of expansion, if the above processes are replaced by a single reversible polytropic process giving the same work between the same initial and final states.

Take c_p *= 1 kJ/kg K,* c_v *= 0.714 kJ/kg K.*

Solution. Refer Fig. 4.22.

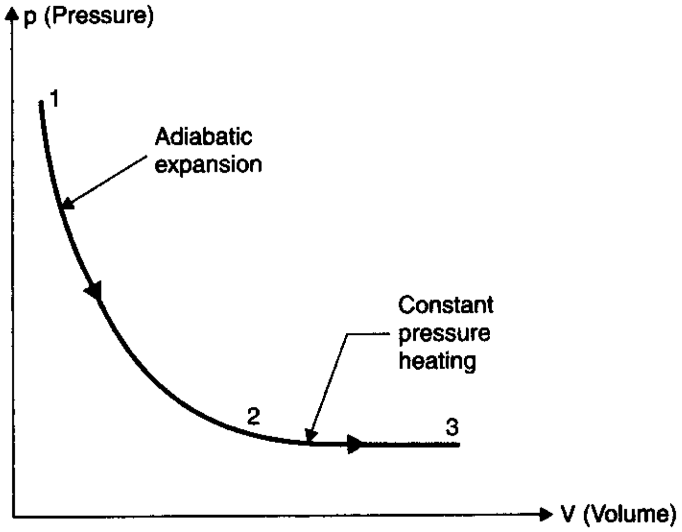


Fig. 4.22

Initial volume, $V_1 = 0.2 \text{ m}^3$

Initial pressure, $p_1 = 4 \text{ bar} = 4 \times 10^5 \text{ N/m}^2$

Initial temperature, $T_1 = 130 + 273 = 403 \text{ K}$

Final pressure after adiabatic expansion,

$$p_2 = 1.02 \text{ bar} = 1.02 \times 10^5 \text{ N/m}^2$$

Increase in enthalpy during constant pressure process

$$= 72.5 \text{ kJ.}$$

(*i*) **Work done :**

Process 1-2 : *Reversibe adiabatic process :*

$$p_1 V_1^{\gamma} = p_2 V_2^{\gamma}$$

$$V_2 = V_1 \left(\frac{p_1}{p_2}\right)^{\frac{1}{\gamma}}$$

Also $$\gamma = \frac{c_p}{c_v} = \frac{1}{0.714} = 1.4$$

$\therefore$ $$V_2 = 0.2 \times \left(\frac{4 \times 10^5}{1.02 \times 10^5}\right)^{\frac{1}{1.4}} = 0.53\ \text{m}^3$$

Also, $$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma - 1}{\gamma}}$$

$\therefore$ $$T_2 = T_1 \left(\frac{p_2}{p_1}\right)^{\frac{\gamma - 1}{\gamma}}$$

$$= 403 \left(\frac{1.02 \times 10^5}{4 \times 10^5}\right)^{\frac{1.4 - 1}{1.4}} = 272.7\ \text{K}$$

Mass of the gas,

$$m = \frac{p_1 V_1}{R T_1} \qquad [\because\ pV = mRT]$$

where, $R = (c_p - c_v) = (1 - 0.714)$ kJ/kg K

$= 0.286$ kJ/kg K $= 286$ J/kg K or 286 Nm/kg K

$\therefore$ $$m = \frac{4 \times 10^5 \times 0.2}{286 \times 403} = 0.694\ \text{kg.}$$

Process 2-3. *Constant pressure* :

$$Q_{2\text{-}3} = mc_p\,(T_3 - T_2)$$

$$72.5 = 0.694 \times 1 \times (T_3 - 272.7)$$

$\therefore$ $$T_3 = \frac{72.5}{0.694} + 272.7 = 377\ \text{K}$$

Also, $$\frac{V_2}{T_2} = \frac{V_3}{T_3}$$

or $$\frac{0.53}{272.7} = \frac{V_3}{377}$$

$\therefore$ $$V_3 = \frac{0.53 \times 377}{272.7} = 0.732\ \text{m}^3$$

Work done by the path 1-2-3 is given by

$$W_{1\text{-}2\text{-}3} = W_{1\text{-}2} + W_{2\text{-}3}$$

$$= \frac{p_1 V_1 - p_2 V_2}{\gamma - 1} + p_2\,(V_3 - V_2)$$

$$= \frac{4 \times 10^5 \times 0.2 - 1.02 \times 10^5 \times 0.53}{1.4 - 1} + 1.02 \times 10^5\,(0.732 - 0.53)$$

$$= \frac{10^5(4 \times 0.2 - 1.02 \times 0.53)}{0.4} + 1.02 \times 10^5\,(0.732 - 0.53)$$

$$= 64850 + 20604 = 85454\ \text{Nm or J}$$

Hence, **total work done** **= 85454 Nm or J. (Ans.)**

(*ii*) **Index of expansion, n :**

If the work done by the polytropic process is the same,

$$W_{1-2-3} = W_{1-3} = \frac{p_1V_1 - p_3V_3}{n-1}$$

$$85454 = \frac{4 \times 10^5 \times 0.2 - 1.02 \times 10^5 \times 0.732}{(n-1)} = \frac{5336}{n-1}$$

$$\therefore \quad n = \frac{5336}{85454} + 1$$

i.e., $n = 1.062$

Hence, **value of index = 1.062. (Ans.)**

Example 4.24. *The following is the equation which connects u, p and v for several gases*

$$u = a + bpv$$

where a and b are constants. Prove that for a reversible adiabatic process,

$$pv^{\gamma} = \text{constant, where } \gamma = \frac{b+1}{b}.$$

Solution. Consider a unit mass.

For a reversible adiabatic process, first law gives

$$0 = du + pdv$$

$$\therefore \quad \frac{du}{dv} = -p \qquad \ldots(i)$$

Also, $u = a + bpv$

$$\therefore \quad \frac{du}{dv} = \frac{d(a+bpv)}{dv} = bv\frac{dp}{dv} + bp$$

$$= b\left(p + v.\frac{dp}{dv}\right) \qquad \ldots(ii)$$

Equating (*i*) and (*ii*), we get

$$b\left(p + v.\frac{dp}{dv}\right) = -p$$

$$bp + b.v.\frac{dp}{dv} = -p$$

$$bp + p + bv.\frac{dp}{dv} = 0$$

$$p(b+1) + bv.\frac{dp}{dv} = 0$$

Multiplying both sides by $\frac{dv}{bpv}$, we get

$$\left(\frac{b+1}{b}\right)\frac{dv}{v} + \frac{dp}{p} = 0$$

or

$$\frac{dp}{p} + \left(\frac{b+1}{b}\right)\frac{dv}{v} = 0$$

$$d(\log_e p) + \left(\frac{b+1}{b}\right) d(\log_e v) = 0$$

Also, $$\frac{b+1}{b} = \gamma \qquad \text{...(Given)}$$

$\therefore \quad d(\log_e p) + \gamma d\,(\log_e v) = 0$

Integrating, we get pv^γ = constant.

Example 4.25. *A 15 cm diameter vertical cylinder, closed by a piston contains a combustible mixture at a temperature of 30°C. The piston is free to move and its weight is such that the mixture pressure is 3 bar. Upper surface of the piston is exposed to the atmosphere. The mixture is ignited. As the reaction proceeds, the piston moves slowly upwards and heat transfer to the surroundings takes place. When the reaction is complete and the contents have been reduced to the initial temperature of 30°C, it is found that the piston has moved upwards a distance of 8.5 cm and the magnitude of heat transfer is 4 kJ. Evaluate :*

(i) The work ;

(ii) Decrease in internal energy of the system.

Solution. Diameter of vertical cylinder, d = 15 cm (or 0.15 m)

Temperature of combustible mixture = 30°C (or 303 K)

Pressure of the mixture $= 3 \text{ bar} = 3 \times 10^5 \text{ N/m}^2$

Upward displacement of the system = 8.5 cm (or 0.085 m)

Magnitude of heat transfer, $Q = -4$ kJ ...(*i*)

(*i*) Work done by the system, $W = \int pdv$

$$= 3 \times 10^5 \int dv \qquad [\because \; p = \text{constant} = 3 \times 10^5 \text{ N/m}^2]$$

$$= 3 \times 10^5 \left[\frac{\pi}{4} \times (0.15)^2 \times 0.085\right] \text{N-m}$$

$$= 450.62 \text{ N-m or J} = 0.4506 \text{ kJ}$$

$$\therefore \quad W = 0.4506 \text{ kJ.}$$

(*ii*) By first law of thermodynamics,

$$Q = \Delta U + W$$

$$-4 = \Delta U + 0.4506$$

$$\therefore \quad \Delta U = -4.4506 \text{ kJ}$$

$\therefore$ **Decrease in internal energy = 4.4506 kJ. (Ans.)**

Example 4.26. *A house wife, on a warm summer day, decides to beat the heat by closing the windows and doors in the kitchen and opening the refrigerator door. At first she feels cool and refreshed, but after a while the effect begins to wear off.*

Evaluate the situation as it relates to First Law of Thermodynamics, considering the room including the refrigerator as the system.

Solution. Initially, the temperature of air in the room falls when it communicates with the cool refrigerator with its door open. This makes the house wife feel cool.

Considering the room and its contents as the system, and assuming the walls, windows and doors non-conducting, we find, $Q = 0$.

To operate the refrigerator, *electricity* is supplied from outside and hence *external work W is done on the system.*

Applying the first law to the system,

$$Q = \Delta U + W$$

$$0 = \Delta U + (-W)$$

$$\therefore \quad \Delta U = W$$

The right hand side is a positive figure indicating the increase in energy of the system with time. As the energy is increasing the temperature of air increases and hence the effect of coolness gradually begins to wear off.

It may be pointed out here that in this case the energy rise manifests itself in a rise in temperature.

☞**Example 4.27.** *A cylinder contains 0.45 m³ of a gas at 1 × 10⁵ N/m² and 80°C. The gas is compressed to a volume of 0.13 m³, the final pressure being 5 × 10⁵ N/m². Determine :*

(i) The mass of gas ;

(ii) The value of index 'n' for compression ;

(iii) The increase in internal energy of the gas ;

(iv) The heat received or rejected by the gas during compression.

Take $\gamma = 1.4$, $R = 294.2$ *J/kg°C.*

Solution. Initial volume of gas, $V_1 = 0.45 \text{ m}^3$

Initial pressure of gas, $p_1 = 1 \times 10^5 \text{ N/m}^2$

Initial temperature, $T_1 = 80 + 273 = 353 \text{ K}$

Final volume after compression, $V_2 = 0.13 \text{ m}^3$

The final pressure, $p_2 = 5 \times 10^5 \text{ N/m}^2$.

(*i*) To find mass '*m*' using the relation

$$m = \frac{p_1V_1}{RT_1} = \frac{1 \times 10^5 \times 0.45}{294.2 \times 353} = \mathbf{0.433\ kg.}\quad \textbf{(Ans.)}$$

(*ii*) To find index '*n*' using the relation

$$p_1V_1^n = p_2V_2^n$$

or
$$\left(\frac{V_1}{V_2}\right)^n = \frac{p_2}{p_1}$$

$$\left(\frac{0.45}{0.13}\right)^n = \left(\frac{5 \times 10^5}{1 \times 10^5}\right) = 5$$

or
$$(3.46)^n = 5$$

Taking log on both sides, we get

$$n \log_e 3.46 = \log_e 5$$

$$n = \log_e 5/\log_e 3.46 = \mathbf{1.296.}\quad \textbf{(Ans.)}$$

(*iii*) In a polytropic process,

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{n-1} = \left(\frac{0.45}{0.13}\right)^{1.296-1} = 1.444$$

$$\therefore \quad T_2 = 353 \times 1.444 = 509.7 \text{ K}$$

Now, increase in internal energy,

$$\Delta U = mc_v (T_2 - T_1)$$

$$= 0.433 \times \frac{R}{(\gamma - 1)} (T_2 - T_1) \qquad \left[\because c_v = \frac{R}{(\gamma - 1)}\right]$$

$$= 0.433 \times \frac{294.2}{(1.4 - 1)\, 1000} (509.7 - 353)$$

$$= \mathbf{49.9\ kJ.}\quad \textbf{(Ans.)}$$

(*iv*) $$Q = \Delta U + W$$

Now, $$W = \frac{p_1V_1 - p_2V_2}{n-1} = \frac{mR(T_1 - T_2)}{n-1}$$

$$= \frac{0.433 \times 294.2(353 - 509.7)}{1.296 - 1}$$

$$= -\,67438 \text{ N-m or } -\,67438 \text{ J} = -\,67.44 \text{ kJ}$$

$$\therefore \quad Q = 49.9 + (-\,67.44) = -\,17.54 \text{ kJ}$$

∴ **Heat rejected = 17.54 kJ. (Ans.)**

Example 4.28. *Air at 1.02 bar, 22°C, initially occupying a cylinder volume of 0.015 m³, is compressed reversibly and adiabatically by a piston to a pressure of 6.8 bar. Calculate :*

(*i*) *The final temperature ;*

(*ii*) *The final volume ;*

(*iii*) *The work done.*

Solution. Initial pressure, $p_1 = 1.02$ bar

Initial temperature, $T_1 = 22 + 273 = 295$ K

Initial volume, $V_1 = 0.015 \text{ m}^3$

Final pressure, $p_2 = 6.8$ bar

Law of compression : $pv^\gamma = C$

(*i*) **Final temperature :**

Using the relation,

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma - 1}{\gamma}}$$

$$\frac{T_2}{295} = \left(\frac{6.8}{1.02}\right)^{\frac{1.4-1}{1.4}} \qquad [\because \ \gamma \text{ for air} = 1.4]$$

$$\therefore \quad T_2 = 295\left(\frac{6.8}{1.02}\right)^{\frac{1.4-1}{1.4}} = 507.24 \text{ K}$$

i.e., **Final temperature** = 507.24 – 273 = **234.24°C. (Ans.)**

(*ii*) **Final volume :**

Using the relation,

$$p_1V_1^\gamma = p_2V_2^\gamma$$

$$\frac{p_1}{p_2} = \left(\frac{V_2}{V_1}\right)^\gamma \quad \text{or} \quad \frac{V_2}{V_1} = \left(\frac{p_1}{p_2}\right)^{\frac{1}{\gamma}}$$

$$\therefore \quad V_2 = V_1 \times \left(\frac{p_1}{p_2}\right)^{\frac{1}{\gamma}} = 0.015 \times \left(\frac{1.02}{6.8}\right)^{\frac{1}{1.4}} = 0.00387 \text{ m}^3$$

i.e., **Final volume = 0.00387 m³. (Ans.)**

Now, work done *on the air*,

$$W = \frac{mR\,(T_1 - T_2)}{(\gamma - 1)} \qquad ...(i)$$

where *m* is the mass of air and is found by the following relation,

$$pV = mRT$$

$$\therefore \quad m = \frac{p_1V_1}{RT_1} = \frac{1.02 \times 10^5 \times 0.015}{0.287 \times 10^3 \times 295} \qquad [\because \; R \text{ for air} = 0.287 \times 10^3]$$

$$= 0.01807 \text{ kg}$$

$$\therefore \quad W = \frac{0.01807 \times 0.287 \times 10^3 (295 - 507.24)}{(1.4 - 1)} = -2751 \text{ J or } -2.751 \text{ kJ}$$

i.e., **Work done = 2.751 kJ. (Ans.)**

(– ve sign indicates that work is done on the air).

Example 4.29. *0.44 kg of air at 180°C expands adiabatically to three times its original volume and during the process, there is a fall in temperature to 15°C. The work done during the process is 52.5 kJ. Calculate c_p and c_v.*

Solution. Refer Fig. 4.23.

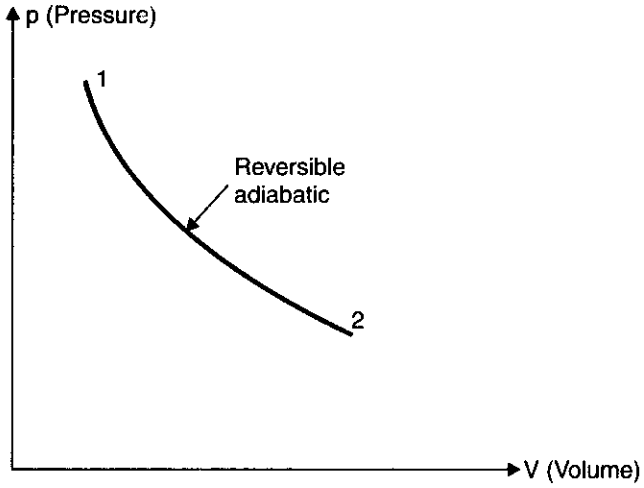


Fig. 4.23

Mass of air, $m = 0.44$ kg

Initial temperature, $T_1 = 180 + 273 = 453$ K

$$\text{Ratio} = \frac{V_2}{V_1} = 3$$

Final temperature, $T_2 = 15 + 273 = 288$ K

Work done during the process, $W_{1-2} = 52.5$ kJ

$$c_p = ?, \; c_v = ?$$

For adiabatic process, we have

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma - 1}$$

$$\frac{288}{453} = \left(\frac{1}{3}\right)^{\gamma - 1} \quad \text{or} \quad 0.6357 = (0.333)^{\gamma - 1}$$

or Taking log on both sides, we get

$$\log_e (0.6357) = (\gamma - 1) \log_e (0.333)$$

$$- 0.453 = (\gamma - 1) \times (- 1.0996)$$

$\therefore$ $$\gamma = \frac{0.453}{1.0996} + 1 = 1.41$$

Also, $$\frac{c_p}{c_v} = \gamma = 1.41$$

Work done during adiabatic process,

$$W_{1-2} = \frac{mR(T_1 - T_2)}{\gamma - 1}$$

$\therefore$ $$52.5 = \frac{0.44\, R(453 - 288)}{(1.41 - 1)}$$

$\therefore$ $$R = \frac{52.5\,(1.41 - 1)}{0.44\,(453 - 288)} = 0.296$$

$\therefore$ $$c_p - c_v = 0.296 \qquad [\because \; R = c_p - c_v]$$

Also $$\frac{c_p}{c_v} = 1.41 \quad \text{or} \quad c_p = 1.41\, c_v$$

$\therefore$ $$1.41\, c_v - c_p = 0.296$$

or c_v = **0.722 kJ/kg K. (Ans.)**

and c_p = **1.018 kJ/kg K. (Ans.)**

☞ **Example 4.30.** *1 kg of ethane (perfect) gas is compressed from 1.1 bar, 27°C according to a law* $pV^{1.3}$ *= constant, until the pressure is 6.6 bar. Calculate the heat flow to or from the cylinder walls.*

Given : Molecular weight of ethane = 30, c_p *= 1.75 kJ/kg K.*

Solution. Mass of ethane gas, $m = 1$ kg

Initial pressure, $p_1 = 1.1$ bar

Initial temperature, $T_1 = 27 + 273 = 300$ K

Final pressure, $p_2 = 6.6$ bar

Law of compression, $pV^{1.3} = C$

Quantity of heat transferred, Q :

Now, characteristic gas constant,

$$R = \frac{\text{Universal gas constart } (R_0)}{\text{Molecular weight } (M)}$$

$$= \frac{8314}{30} = 277.13 \text{ N-m/kg K} = 277.31 \text{ J/kg K}$$

$$= 0.277 \text{ kJ/kg K}$$

Also $$c_p - c_v = R$$

$\therefore$ $$c_v = c_p - R = 1.75 - 0.277 = 1.473 \text{ kJ/kg K}$$

$$\gamma = \frac{c_p}{c_v} = \frac{1.75}{1.473} = 1.188$$

In case of a polytropic process,

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} = \left(\frac{6.6}{1.1}\right)^{\frac{1.3-1}{1.3}} = 1.5119$$

$$\therefore \quad T_2 = 300 \times 1.5119 = 453.6 \text{ K}$$

Now, work done, $W = \frac{R(T_1 - T_2)}{n-1} = \frac{0.277(300 - 453.6)}{1.3 - 1} = -\ 141.8 \text{ kJ/kg}$

To find heat flow, using the relation,

$$Q = \left(\frac{\gamma - n}{\gamma - 1}\right) W = \left(\frac{1.188 - 1.3}{1.188 - 1}\right) \times -\ 141.8 = +\ 84.5 \text{ kJ/kg}$$

i.e., **Heat supplied = 84.5 kJ/kg. (Ans.)**

Example 4.31. *0.1 m^3 of an ideal gas at 300 K and 1 bar is compressed adiabatically to 8 bar. It is then cooled at constant volume and further expanded isothermally so as to reach the condition from where it started. Calculate :*

(i) Pressure at the end of constant volume cooling.

(ii) Change in internal energy during constant volume process.

(iii) Net work done and heat transferred during the cycle. Assume

c_p = 14.3 kJ/kg K and c_v = 10.2 kJ/kg K.

Solution. *Given :* $V_1 = 0.1 \text{ m}^3$; $T_1 = 300$ K ; $p_1 = 1$ bar ; $c_p = 14.3$ kJ/kg K ; $c_v = 10.2$ kJ/kg K.

Refer to Fig. 4.24.

(*i*) **Pressure at the end of constant volume cooling, p_3 :**

$$\gamma = \frac{c_p}{c_v} = \frac{14.3}{10.2} = 1.402$$

Characteristic gas constant,

$$R = c_p - c_v = 14.3 - 10.2 = 4.1 \text{ kJ/kg K}$$

Considering *process 1-2*, we have :

$$p_1V_1^{\gamma} = p_2V_2^{\gamma}$$

$$V_2 = V_1 \times \left(\frac{p_1}{p_2}\right)^{\frac{1}{\gamma}} = 0.1 \times \left(\frac{1}{8}\right)^{\frac{1}{1.402}} = 0.0227 \text{ m}^3$$

Also, $\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{8}{1}\right)^{\frac{1.402-1}{1.402}} = 1.815$

or $\quad T_2 = T_1 \times 1.815 = 300 \times 1.815 = 544.5$ K

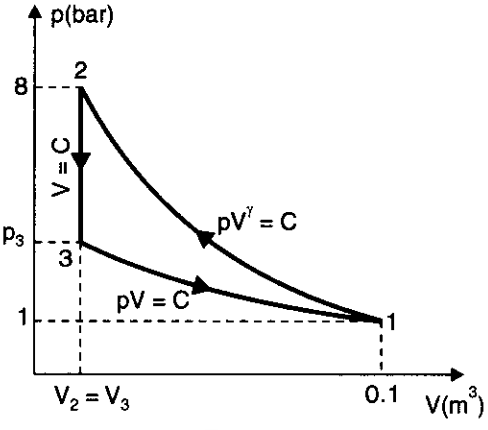


Fig. 4.24

Considering *process 3–1*, we have

$$p_3V_3 = p_1V_1$$

$$\therefore \quad p_3 = \frac{p_1V_1}{V_3} = \frac{1 \times 0.1}{0.0227} = \textbf{4.4 bar. (Ans.)} \qquad (\because \ V_3 = V_2)$$

(*ii*) **Change in internal energy during constant volume process, $(U_3 - U_2)$:**

Mass of gas, $\quad m = \frac{p_1V_1}{RT_1} = \frac{(1 \times 10^5) \times 0.1}{(4.1 \times 1000) \times 300} = 0.00813$ kg

∴ Change in internal energy during *constant volume process 2–3,*

$$U_3 - U_2 = mc_v(T_3 - T_2)$$
$$= 0.00813 \times 10.2\,(300 - 544.5) \qquad (\because\ T_3 = T_1)$$
$$= \mathbf{-\ 20.27\ kJ.}\ \ \textbf{(Ans.)}$$

(– ve sign means *decrease* in internal energy)

- During constant volume cooling process, temperature and hence internal energy is *reduced.* This *decrease in internal energy equals to heat flow to surroundings since work done is zero.*

(*iii*) **Net work done and heat transferred during the cycle :**

$$W_{1-2} = \frac{p_1V_1 - p_2V_2}{\gamma - 1} = \frac{mR(T_1 - T_2)}{\gamma - 1}$$
$$= \frac{0.00813 \times 4.1(300 - 544.5)}{1.402 - 1} = -\ 20.27\ \text{kJ}$$

$W_{2-3} = 0$ ... since volume remains constant

$$W_{3-1} = p_3V_3 \log_e \left(\frac{V_1}{V_3}\right) = p_1V_1 \log_e \left(\frac{p_3}{p_1}\right) \qquad (\because\ p_3V_3 = p_1V_1)$$
$$= (1 \times 10^5) \times 0.1 \times \log_e \left(\frac{4.4}{1}\right)$$
$$= 14816\ \text{Nm (or J) or } 14.82\ \text{kJ}$$

∴ **Net work done** $= W_{1-2} + W_{2-3} + W_{3-1}$

$= (-\ 20.27) + 0 + 14.82 = \mathbf{-\ 5.45\ kJ}$

–ve sign indicates that work has been done *on the system.* **(Ans.)**

For a cyclic process : $\oint \delta Q = \oint \delta W$

∴ Heat transferred during the complete cycle = – **5.45 kJ**

–ve sign means heat has been *rejected i.e., lost from the system.* **(Ans.)**

Example 4.32. *0.15 m^3 of an ideal gas at a pressure of 15 bar and 550 K is expanded isothermally to 4 times the initial volume. It is then cooled to 290 K at constant volume and then compressed back polytropically to its initial state.*

Calculate the net work done and heat transferred during the cycle.

Solution. *Given* : $V_1 = 0.15\ \text{m}^3$; $p_1 = 15$ bar ; $T_1 = T_2 = 550$ K ; $\frac{V_2}{V_1} = 4$; $T_3 = 290$ K

Refer to Fig. 4.25.

Considering the *isothermal process* 1–2, we have

$$p_1V_1 = p_2V_2 \quad \text{or} \quad p_2 = \frac{p_1V_1}{V_2}$$

or,
$$p_2 = \frac{15 \times 0.15}{(4 \times 0.15)} = 3.75\ \text{bar}$$

Work done, $W_{1-2} = p_1V_1 \log_e \left(\frac{V_2}{V_1}\right)$

$= (15 \times 10^5) \times 0.15 \times \log_e (4)$

$= 311916 \text{ J} = 311.9 \text{ kJ}$

Considering *constant volume process 2–3*, we get

$$V_2 = V_3 = 4 \times 0.15 = 0.6 \text{ m}^3$$

$$\frac{p_2}{T_2} = \frac{p_3}{T_3}$$

or, $$p_3 = p_2 \times \frac{T_3}{T_2} = 3.75 \times \frac{290}{550} = 1.98 \text{ bar}$$

$W_{2-3} = 0$

... since volume remains constant

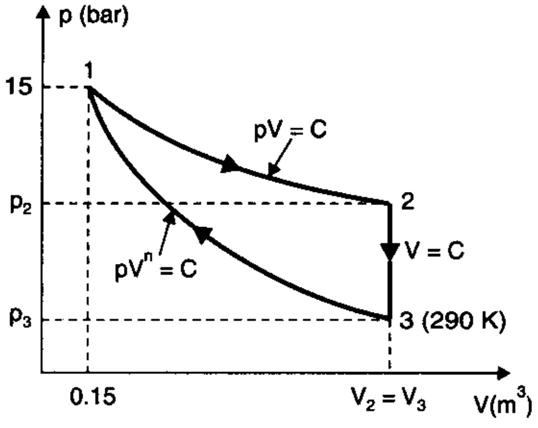


Fig. 4.25

Consider *polytropic process 3–1 :*

$$p_3V_3^n = p_1V_1^n \quad \text{or} \quad \frac{p_1}{p_3} = \left(\frac{V_3}{V_1}\right)^n$$

Taking log on both sides, we get

$$\log_e (p_1/p_3) = n \log_e (V_3/V_1)$$

or, $$n = \frac{\log_e(p_1/p_3)}{\log_e(V_3/V_1)} = \frac{\log_e (15/1.98)}{\log_e (4)} = 1.46$$

$$W_{3-1} = \frac{p_3V_3 - p_1V_1}{n-1} = \frac{1.98 \times 10^5 \times 0.6 - 15 \times 10^5 \times 0.15}{(1.46 - 1)}$$

$= - 230869 \text{ J} \quad \text{or} \quad - 230.87 \text{ kJ}$

$\therefore$ *Net work done* $= W_{1-2} + W_{2-3} + W_{3-1}$

$= 311.9 + 0 + (- 230.87) =$ **81.03 kJ. (Ans.)**

For a cyclic process, $\oint \delta Q = \oint \delta W$

$\therefore$ Heat *transferred during the cycle* = **81.03 kJ. (Ans.)**

Example 4.33. *A system consisting of 1 kg of an ideal gas at 5 bar pressure and 0.02 m³ volume executes a cyclic process comprising the following three distinct operations : (i) Reversible expansion to 0.08 m³ volume, 1.5 bar pressure, presuming pressure to be a linear function of volume (p = a + bV), (ii) Reversible cooling at constant pressure and (iii) Reversible hyperbolic compression according to law pV = constant. This brings the gas back to initial conditions.*

(i) Sketch the cycle on p-V diagram.

(ii) Calculate the work done in each process starting whether it is done on or by the system and evaluate the net cyclic work and heat transfer.

Solution. *Given :* $m = 1$ kg ; $p_1 = 5$ bar ; $V_1 = 0.02$ m³ ; $V_2 = 0.08$ m³ ; $p_2 = 1.5$ bar.

(*i*) **p-V diagram :** *p-V* diagram of the cycle is shown in Fig. 4.26.

(*ii*) **Work done and heat transfer :**

- *Process 1-2 (Linear law) :*

$$p = a + bV \qquad ...(Given)$$

The values of constants *a* and *b* can be determined from the values of pressure and volume at the state points 1 and 2.

$$5 = a + 0.02b \qquad ...(i)$$

$$1.5 = a + 0.08b \qquad ...(ii)$$

From (*i*) and (*ii*) we get, $b = -58.33$ and $a = 6.167$

$$W_{1-2} = \int_1^2 pdV = \int_1^2 (a + bV)\,dV$$

$$= \int_1^2 (6.167 - 58.33V)\,dV$$

$$= 10^5 \left| 6.167\,V - 58.33 \times \frac{V^2}{2} \right|_{0.02}^{0.08}$$

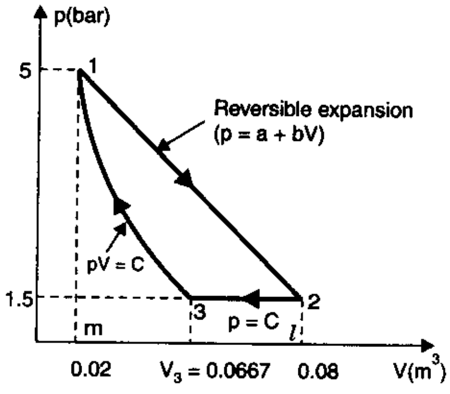


Fig. 4.26. *p-V* diagram.

$$= 10^5 \left| 6.167\,(0.08 - 0.02) - 58.33 \times \frac{(0.08^2 - 0.02^2)}{2} \right| \times 10^{-3}\ \text{kJ} = \mathbf{19.5\ kJ}$$

This is work done **by the system. (Ans.)**

[*Alternatively :* W_{1-2} = Area under the process line 1–2
= Area of trapezium 1–2–*l*–*m*

$$= \left[\frac{5 + 1.5}{2} \times 10^5\right] \times (0.08 - 0.02) = 19.5\ \text{kJ}$$]

- *Process 2 – 3 (constant pressure) :*

$$p_3 = p_2 = 1.5\ \text{bar}$$

The volume V_3 can be worked out from the hyperbolic compression 3–1, as follows :

$$p_1V_1 = p_3V_3 \quad \text{or} \quad V_3 = \frac{p_1V_1}{p_3} = \frac{5 \times 0.02}{1.5} = 0.0667\ \text{m}^3$$

$$\therefore \qquad W_{2-3} = p_2(V_3 - V_2) = 1.5 \times 10^5\,(0.0667 - 0.08) \times 10^{-3}\ \text{kJ} = -\,1.995\ \text{kJ}$$

- *Process 3 – 1 (hyperbolic process) :*

$$W_{3-1} = p_3V_3 \log_e \left(\frac{V_1}{V_3}\right)$$

$$= (10^5 \times 1.5) \times 0.0667 \log_e \left(\frac{0.02}{0.0667}\right) \times 10^{-3}\ \text{kJ} = \mathbf{-\,12.05\ kJ.}$$

This is the work **done on the system. (Ans.)**

Net work done, $W_{\text{net}} = W_{1-2} + W_{2-3} + W_{3-1}$

$= 19.5 + (-\,1.995) + (-\,12.05) = \mathbf{5.445\ kJ.}$ **(Ans.)**

Heat transferred during the complete cycle, $\oint \delta Q = \oint \delta W$ = **5.455 kJ. (Ans.)**

Example 4.34. *Fig. 4.27 shows a cylinder of 8 cm inside diameter having a piston loaded with a spring (stiffness = 150 N/cm of compression). The initial pressure, volume and temperature of air in the cylinder are 3 × 10^5 N/m², 0.000045 m³ and 20°C respectively. Determine the amount of heat added to the system so that piston moves by 3.5 cm.*

Assume $c_v = 0.71$ *kJ/kg K and* $R = 0.287$ *kJ/kg K.*

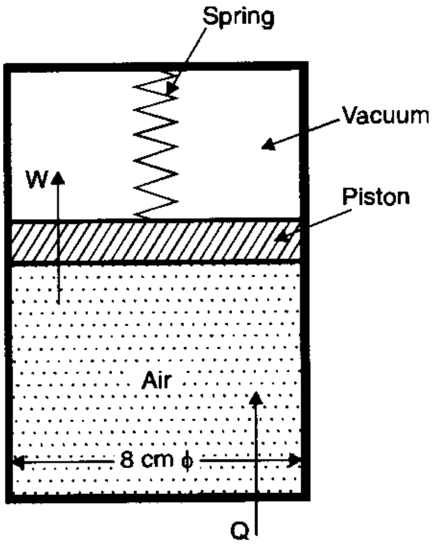


Fig. 4.27

Solution. Insider diameter of the cylinder

= 8 cm

Stiffness of the spring, $S = 150$ N/cm

Initial pressure of air,

$p_1 = 3 \times 10^5$ N/m^2 or 30 N/cm^2

Initial volume of air,

$V_1 = 0.000045$ m^3 = 45 cm^3

Initial temperature of air,

$T_1 = 20 + 273 = 293$ K

Specific heat at constant volume,

$c_v = 0.71$ kJ/kg K

Characteristic constant for air,

$R = 0.287$ kJ/kg K

Refer Fig. 4.28.

Let, oo = An arbitrary datum from which the position of the lower face of the piston is to be measured,

y = Distance of the lower face of the piston,

$y = y_0$, when spring length is its free length, and

p = Pressure of air within the cylinder when $y = y_0$.

Now, force balance for the piston is given by

$$Ap = S(y - y_0) \quad ...(i)$$

where, A = The area of the piston, and

S = Stiffness of the spring.

With heat transfer to the air, let the pressure inside the cylinder increase by dp forcing the piston to move upward by distance dy. Now the force balance for the piston is

$$A(p + dp) = S(y + dy - y_0) \quad ...(ii)$$

From eqns. (*i*) and (*ii*), we have

$$Adp = Sdy \quad ...(iii)$$

The increase in volume dV of the gas for the piston displacement is given by

$$dV = Ady \quad ...(iv)$$

$$\therefore \quad dp = \frac{S}{A^2} dy \quad ...(v)$$

$$\therefore \quad p = \frac{S}{A^2} V + C \quad ...(vi)$$

The p-V relationship for the process is a straight line (Fig. 4.29) having a slope of $\frac{S}{A^2}$ and pressure axis intercept of C. The value of C can be found out from the knowledge of pressure and volume at any state point.

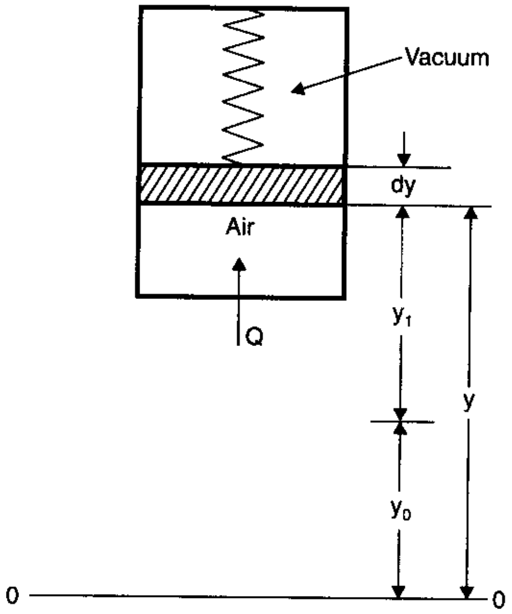


Fig. 4.28

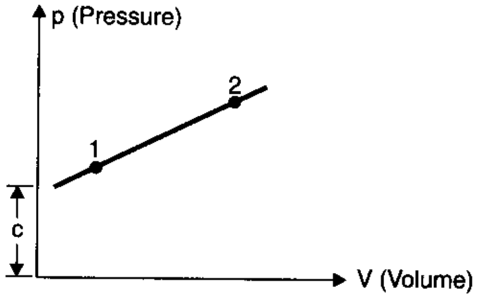


Fig. 4.29

Now, substituting the values of p_1, V_1, A in eqn. (*vi*), we get

$$p = \frac{150}{\left(\frac{\pi}{4} \times 8^2\right)^2} V + C$$

or
$$p = 0.0594\ V + C \quad \text{...(vii)}$$

where p is in N/cm^2 and V is in cm^3.

$$\therefore \quad p_1 = 0.0594\ V_1 + C$$

$$30 = 0.0594 \times 45 + C$$

$$\therefore \quad C = 27.33$$

Hence, p-V relationship for the process is,

$$p = 0.0594\ V + 27.33 \quad \text{...(viii)}$$

During the process the piston is moved by a distance of 3.5 cm. This increases the volume of gas by

$$3.5 \times A^2 = 3.5 \times \left(\frac{\pi}{4} \times 8^2\right) = 175.9 \text{ cm}^3$$

Hence, the final volume of air,

$$V_2 = 45 + 175.9 = 220.9 \text{ cm}^3$$

Substituting this value in equation (*viii*), we get

$$p(= p_2) = 0.0594 \times 220.9 + 27.33 = 40.45 \text{ N/cm}^2$$

The work done W during the process is given by

$$W = \int p dV = \int_{p_1}^{p_2} \frac{A^2}{S} p dp$$

$$= \frac{A^2}{S}\left(\frac{p_2^{\,2} - p_1^{\,2}}{2}\right) = \frac{A^2}{S}\left(\frac{p_2 + p_1}{2}\right)\left(\frac{p_2 - p_1}{2}\right)$$

$$= \frac{A^2}{S}\left(\frac{p_2 + p_1}{2}\right)\frac{S}{A^2}(V_2 - V_1)$$

or $$W = \left(\frac{p_2 + p_1}{2}\right)(V_2 - V_1) \quad ...(ix)$$

= (Mean pressure) × (Change in volume)

$$W = \left(\frac{40.45 + 30}{2}\right) \times (220.9 - 45)$$

= 6196 N-cm or 61.96 N-m

It may be noted that work done does not cross the system boundary when spring and cylinder are considered system.

Now, to find T_2, using the relation,

$$\frac{p_1V_1}{T_1} = \frac{p_2V_2}{T_2}$$

$$\therefore \quad T_2 = \frac{p_2V_2T_1}{p_1V_1} = \frac{40.45 \times 220.9 \times 293}{30 \times 45} = 1939.3 \text{ K}$$

Also, $$m = \frac{p_1v_1}{R_1T_1} = \frac{30 \times 45}{(0.287 \times 10^3) \times 293} = 0.0001605 \text{ kg}$$

Now, change in internal energy,

$$\Delta U = m \times c_v \times (T_2 - T_1)$$

$$= 0.0001605 \times 0.71 \times (1939.3 - 293) = 0.1876 \text{ kJ}$$

According to first law,

$$Q_{1-2} = \Delta U + W$$

$$= 0.1876 + 61.96 \times 10^{-3} = 0.2495 \text{ kJ}$$

∴ **Amount of heat added to the system = 0.2495 kJ. (Ans.)**

4.10. APPLICATION OF FIRST LAW TO STEADY FLOW PROCESS

Steady Flow Energy Equation (S.F.E.E.)

In many practical problems, the rate at which the fluid flows through a machine or piece of apparatus is constant. This type of flow is called *steady flow*.

Assumptions :

The following *assumptions* are made in the system analysis :

(*i*) The mass flow through the system remains constant.

(*ii*) Fluid is uniform in composition.

(*iii*) The only interaction between the system and surroundings are work and heat.

(*iv*) The state of fluid at any point remains constant with time.

(*v*) In the analysis only potential, kinetic and flow energies are considered.

Fig. 4.30 shows a schematic flow process for an open system. An open system is one in which both mass and energy may cross the boundaries. A wide interchange of energy may take place within an open system. Let the system be an automatic engine with the inlet manifold at the first state point and exhaust pipe as the second point. There would be an interchange of chemical energy in the fuel, kinetic energy of moving particles, internal energy of gas and heat transferred and shaft work within the system. From Fig. 4.30 it is obvious that if there is no variation of flow of mass or energy with time across the boundaries of the system the steady flow will prevail. The conditions may pass through the cyclic or non-cyclic changes within the system. As a result the mass entering the system equals the mass leaving, also energy entering the system equals energy leaving.

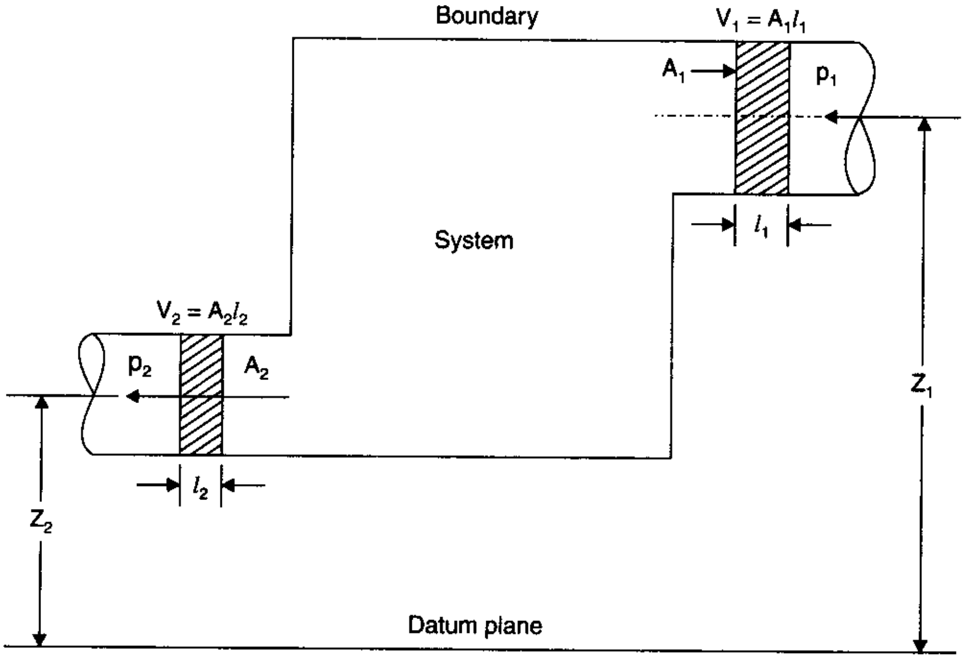


Fig. 4.30

The steady flow equation can be expressed as follows :

$$u_1 + \frac{C_1^2}{2} + Z_1 g + p_1 v_1 + Q = u_2 + \frac{C_2^2}{2} + Z_2 g + p_2 v_2 + W \qquad \text{...(4.45)}$$

$$(u_1 + p_1 v_1) + \frac{C_1^2}{2} + Z_1 g + Q = (u_2 + p_2 v_2) + \frac{C_2^2}{2} + Z_2 g + W$$

$$h_1 + \frac{C_1^2}{2} + Z_1 g + Q = h_2 + \frac{C_2^2}{2} + Z_2 g + W \qquad [\because \; h = u + pv]$$

If Z_1 and Z_2 are neglected, we get

$$h_1 + \frac{C_1^2}{2} + Q = h_2 + \frac{C_2^2}{2} + W \qquad \text{...[4.45 (a)]}$$

where, Q = Heat supplied (or entering the boundary) per kg of fluid,

W = Work done by (or work coming out of the boundary) 1 kg of fluid,

C = Velocity of fluid ,
Z = Height above datum,
p = Pressure of the fluid,
u = Internal energy per kg of fluid, and
pv = Energy required for 1 kg of fluid.

This equation is applicable to any medium in any steady flow. It is applicable not only to rotary machines such as centrifugal fans, pumps and compressors but also to reciprocating machines such as steam engines.

In a steady flow the rate of mass flow of fluid at any section is the same as at any other section. Consider any section of cross-sectional area A, where the fluid velocity is C, the rate of volume flow past the section is CA. Also, since mass flow is volume flow divided by specific volume,

$$\therefore \quad \text{Mass flow rate, } \dot{m} = \frac{CA}{v} \qquad ...(4.46)$$

(where v = *Specific volume* at the section)

This equation is known as the **continuity of mass equation.**

With reference to Fig. 4.30.

$$\therefore \quad \dot{m} = \frac{C_1A_1}{v_1} = \frac{C_2A_2}{v_2} \qquad ...[4.46\ (a)]$$

4.11. ENERGY RELATIONS FOR FLOW PROCESS

The energy equation (m kg of fluid) for a steady flow system is given as follows :

$$m\left(u_1 + \frac{C_1^2}{2} + Z_1g + p_1v_1\right) + Q = m\left(u_2 + \frac{C_2^2}{2} + Z_2g + p_2v_2\right) + W$$

i.e.,
$$Q = m\left[(u_2 - u_1) + (Z_2g - Z_1g) + \left(\frac{C_2^2}{2} - \frac{C_1^2}{2}\right) + (p_2v_2 - p_1v_1)\right] + W$$

i.e.,
$$Q = m\left[(u_2 - u_1) + g(Z_2 - Z_1) + \left(\frac{C_2^2 - C_1^2}{2}\right) + (p_2v_2 - p_1v_1)\right] + W$$

$$= \Delta U + \Delta PE + \Delta KE + \Delta\,(pv) + W$$

where
$$\Delta U = m\,(u_2 - u_1)$$
$$\Delta PE = mg\,(Z_2 - Z_1)$$
$$\Delta KE = m\left(\frac{C_2^2 - C_1^2}{2}\right)$$
$$\Delta pv = m(p_2v_2 - p_1v_1)$$

$$\therefore \quad Q - \Delta U = [\Delta PE + \Delta KE + \Delta(pV) + W] \qquad ...(4.47)$$

For non-flow process,

$$Q = \Delta U + W = \Delta U + \int_1^2 pdV$$

i.e.,
$$Q - \Delta U = \int_1^2 p.dV \qquad ...(4.48)$$

The internal energy is a function of temperature only and it is a point function. Therefore, for the same two temperatures, change in internal energy is the same whatever may be the process, non-flow, or steady flow, reversible or irreversible.

For the same value of Q transferred to non-flow and steady flow process and for the same temperature range, we can equate the values of eqns. (4.47) and (4.48) for $(Q - \Delta U)$.

$$\therefore \qquad \int_1^2 p.dV = \Delta\, PE + \Delta\, KE + \Delta\,(pV) + W \qquad \text{...(4.49)}$$

where, W = Work transfer in flow process

and $\int_1^2 p.dV$ = Total change in mechanical energy of reversible steady flow process.

Property Relations for Energy Equations

We know that

$$h = u + pv$$

Differentiating above equation

$$dh = du + pdv + vdp$$

But $dQ = du + p.dv$ (as per first law applied to closed system)

or $du = dQ - p.dv$

Substituting this value of du in the above equation, we get

$$dh = dQ - p.dv + pdv + vdp$$

$$= dQ + vdp$$

$$\therefore \qquad vdp = dh - dQ$$

$$\therefore \qquad -\int_1^2 vdp = Q - \Delta h \qquad \text{...(4.50)}$$

where $-\int_1^2 vdp$ represents on a p-v diagram the area behind 1-2 as shown in Fig. 4.31 (*b*).

The eqn. (4.47) for a unit mass flow can be written as

$$dQ = d(PE) + d(KE) + du + d(pv) + dW$$

Substituting the value of $dQ = du + p.dv$ in the above equation, we get

$$du + pdv = d(PE) + d(KE) + du + pdv + vdp + dW$$

$$\therefore \qquad -vdp = d(PE) + d(KE) + dW$$

$$\therefore \qquad -\int_1^2 vdp = \Delta\, PE + \Delta\, KE + W \qquad \text{...[4.50 (a)]}$$

If $\Delta PE = 0$ (as in most of thermodynamic systems)

$$-\int_1^2 vdp = \Delta\, KE + W \qquad \text{...[4.50 (b)]}$$

If $W = 0$, the area behind the curve represents $\Delta\, KE$ and if $\Delta\, KE = 0$, area behind the curve represents W which is shaft work.

$-\int_1^2 vdp$ is a *positive quantity and represents work done by the system.*

If $\Delta\, PE = 0$ and $W = 0$, then

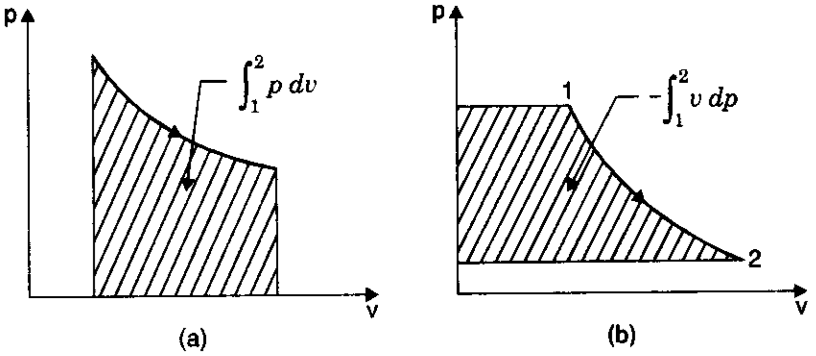
$-\int_1^2 vdp = \Delta\, KE$, this is applicable in case of a *nozzle.*

i.e., $\int_1^2 v dp = \frac{C^2}{2}$ in the case of a nozzle.

If $\Delta PE = 0$ and $\Delta KE = 0$, as in case of a *compressor*, $-\int_1^2 vdp = W$

or $W = \int_1^2 vdp$ in the case of a *compressor*.

The integral $\int_1^2 pdv$ and $\int_1^2 vdp$ are shown in Fig. 4.31 (*a*) and (*b*).



(*a*) Work done in non-flow process. (*b*) Work done in flow process.

Fig. 4.31. Representation of work on *p-v* diagram.

The work done during *non-flow process* is given by

$$\int_1^2 pdv = Q - \Delta u \qquad \text{...[4.50 (c)]}$$

For isothermal process, we have

$$\Delta u = 0 \text{ and } \Delta h = 0.$$

Substituting these values in (equations) 4.50 and [4.50 (*c*)]

$$-\int_1^2 vdp = Q \text{ and } \int_1^2 pdv = Q$$

$$\therefore \quad \int_1^2 pdv = -\int_1^2 vdp$$

The above equation indicates that the *area under both curves is same for an isothermal process.*

Note. In all the above equations '*v*' represents volume per unit mass as mass flow is considered unity.

Now let us find out expressions for work done for different flow processes as follows :

(*i*) **Steady flow constant pressure process :**

$$W = -\int_1^2 v\,.dp = 0 \qquad [\because \; dp = 0] \qquad \text{...(4.51)}$$

(*ii*) **Steady flow constant volume process :**

$$W = -\int_1^2 Vdp = -V(p_2 - p_1) = V(p_1 - p_2)$$

i.e.,
$$W = V(p_1 - p_2) \qquad \text{...(4.52)}$$

(*iii*) **Steady flow constant temperature process :**

The constant temperature process is represented by

$$pV = p_1V_1 = p_2V_2 = C \text{ (constant)}$$

$$\therefore \quad W = -\int_1^2 Vdp$$

$$= -\int_1^2 \frac{C}{p} dp \qquad \left[\because V = \frac{C}{p}\right]$$

$$= -C\int_1^2 \frac{dp}{p} = -C \left| \log_e p \right|_1^2$$

$$= -C \log_e \frac{p_2}{p_1} = C \log_e \frac{p_1}{p_2}$$

i.e.,
$$W = p_1V_1 \log_e \left(\frac{p_1}{p_2}\right) \qquad \text{...(4.53)}$$

Now substituting the values of W in the equation (4.49), considering unit mass flow :

(*a*) The energy equation for *constant pressure flow process*

$$dQ = \Delta PE + \Delta KE + \Delta h$$
$$= \Delta h \text{ (if } \Delta PE = 0 \text{ and } \Delta KE = 0).$$

(*b*) The energy equation for constant volume flow process

$$dQ = -\int_1^2 vdp + \Delta PE + \Delta KE + \Delta u + pdv + vdp$$

$$= \Delta PE + \Delta KE + \Delta u \qquad \left[\because pdv = 0 \text{ and } v \,.\, dp = \int_1^2 vdp\right]$$

$$\therefore \quad dQ = \Delta u \text{ (if } \Delta PE = 0 \text{ and } \Delta KE = 0)$$

4.12. ENGINEERING APPLICATIONS OF STEADY FLOW ENERGY EQUATION (S.F.E.E.)

4.12.1. Water Turbine

Refer to Fig. 4.32. In a water turbine, water is supplied from a height. The potential energy of water is converted into kinetic energy when it enters into the turbine and part of it is converted into useful work which is used to generate electricity.

Considering centre of turbine shaft as *datum*, the energy equation can be written as follows :

$$\left(u_1 + p_1v_1 + Z_1g + \frac{C_1^2}{2}\right) + Q = \left(u_2 + p_2v_2 + Z_2g + \frac{C_2^2}{2}\right) + W$$

In this case,

$$Q = 0$$
$$\Delta u = u_2 - u_1 = 0$$
$$v_1 = v_2 = v$$
$$Z_2 = 0$$

$$\therefore \qquad \left(p_1 v + Z_1 g + \frac{C_1^2}{2}\right) = \left(p_2 v + Z_2 g + \frac{C_2^2}{2}\right) + W \qquad \text{...(4.54)}$$

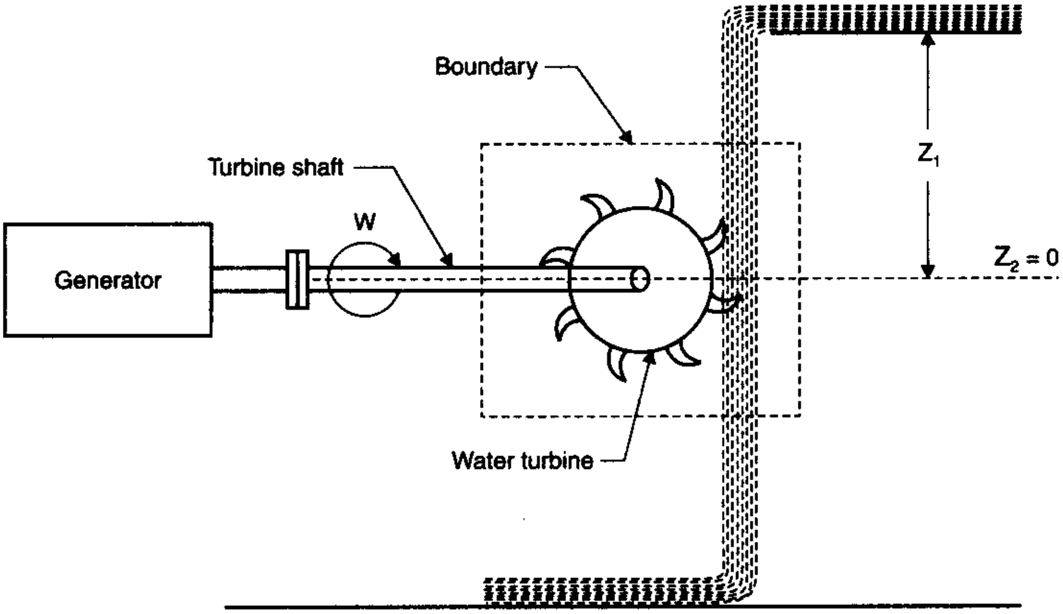


Fig. 4.32. Water turbine.

W is *positive* because work is done by the system (or work comes out of the boundary).

4.12.2. Steam or Gas Turbine

In a steam or gas turbine steam or gas is passed through the turbine and part of its energy is converted into work in the turbine. This output of the turbine runs a generator to produce electricity as shown in Fig. 4.33. The steam or gas leaves the turbine at lower pressure or temperature.

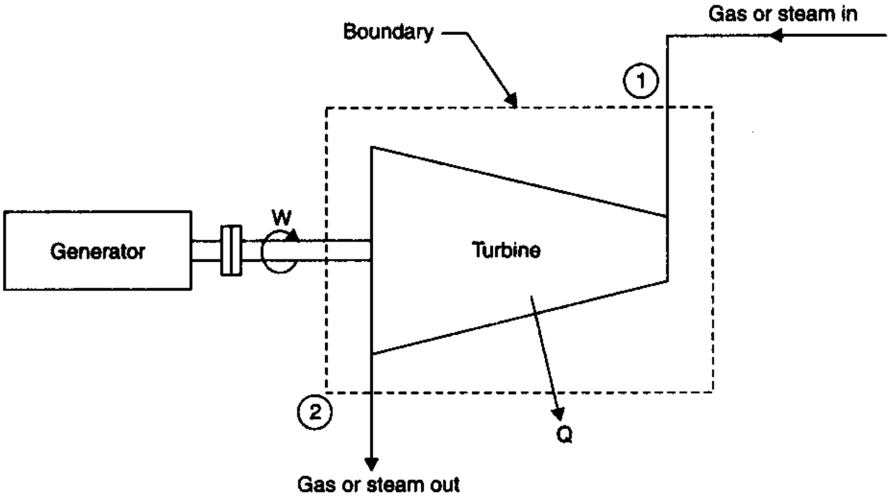


Fig. 4.33. Steam or gas turbine.

Applying energy equation to the system.

Here, $Z_1 = Z_2$ (*i.e.*, $\Delta Z = 0$)

$$h_1 + \frac{C_1^2}{2} - Q = h_2 + \frac{C_2^2}{2} + W \quad \text{...(4.55)}$$

The sign of Q is *negative* because heat is *rejected* (or comes out of the boundary).

The sign of W is *positive* because work is done by the system (or work comes out of the boundary).

4.12.3. Centrifugal Water Pump

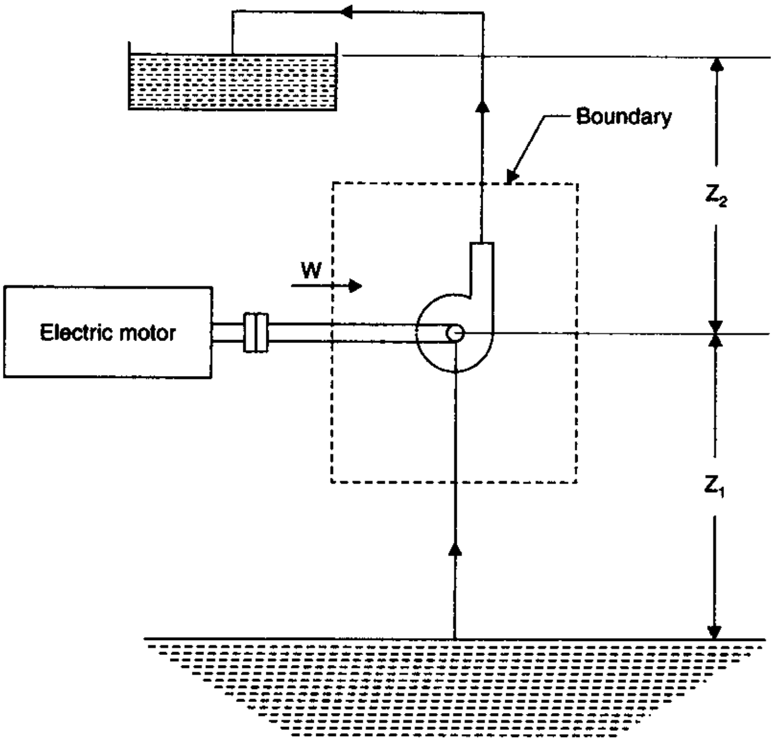


Fig. 4.34. Centrifugal water pump.

A centrifugal water pump draws water from a lower level and pumps to higher level as shown in Fig. 4.34. Work is required to run the pump and this may be supplied from an external source such as an electric motor or a diesel engine.

Here $Q = 0$ and $\Delta u = 0$ as there is no change in temperature of water ; $v_1 = v_2 = v$.

Applying the energy equation to the system

or
$$p_1v_1 + Z_1g + \frac{C_1^2}{2} = p_2v_2 + Z_2g + \frac{C_2^2}{2} - W \quad \text{...(4.56)}$$

The sign of W is *negative* because *work is done on the system* (or *work enters the boundary*).

4.12.4. Centrifugal Compressor

Refer to Fig. 4.35. A centrifugal compressor compresses air and supplies the same at moderate pressure and in large quantity.

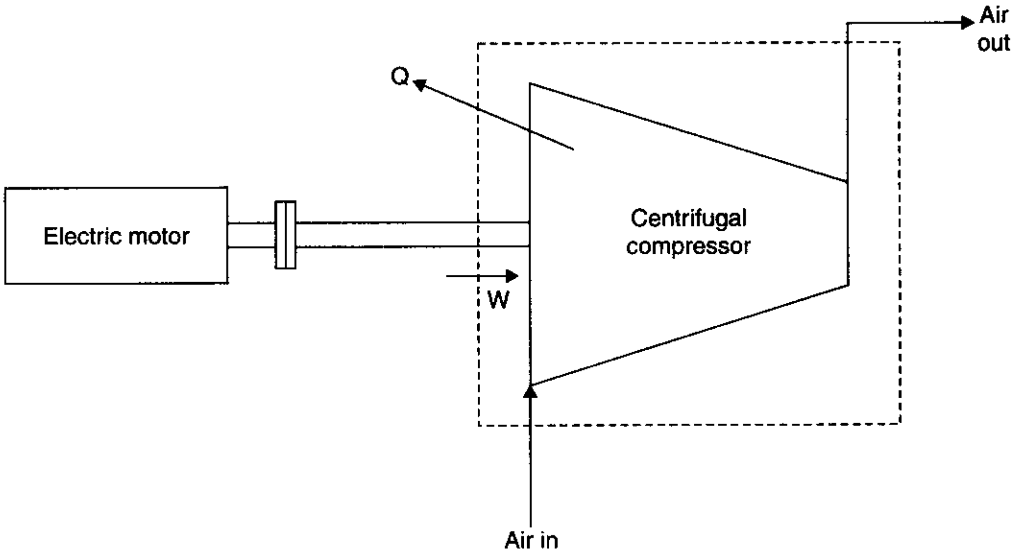


Fig. 4.35. Centrifugal compressor.

Applying energy equation to the system (Fig. 4.35)

$$\Delta Z = 0 \text{ (generally taken)}$$

$$\left(h_1 + \frac{C_1^2}{2}\right) - Q = \left(h_2 + \frac{C_1^2}{2}\right) - W$$

The Q is taken as *negative* as heat is *lost* from the system and W is taken as *negative* as work is *supplied* to the system.

or
$$\left(h_1 + \frac{C_1^2}{2}\right) - Q = \left(h_2 + \frac{C_2^2}{2}\right) - W \qquad ...(4.57)$$

4.12.5. Reciprocating Compressor

Refer Fig. 4.36. The reciprocating compressor draws in air from atmosphere and supplies at a considerable higher pressure in small quantities (compared with centrifugal compressor). The reciprocating compressor can be considered as steady flow system *provided the control volume includes the receiver which reduces the fluctuations of flow considerably.*

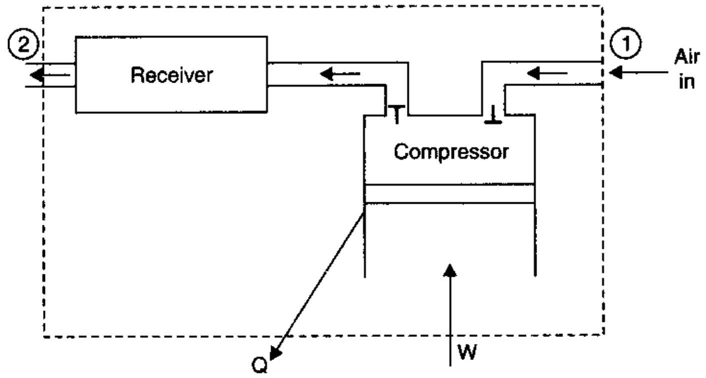


Fig. 4.36. Reciprocating compressor.

Applying energy equation to the system, we have :

$\Delta PE = 0$ and $\Delta KE = 0$ since these changes are negligible compared with other energies.

$$\therefore \qquad h_1 - Q = h_2 - W \qquad \text{...(4.58)}$$

4.12.6. Boiler

A boiler transfers heat to the incoming water and generates the steam. The system is shown in Fig. 4.37.

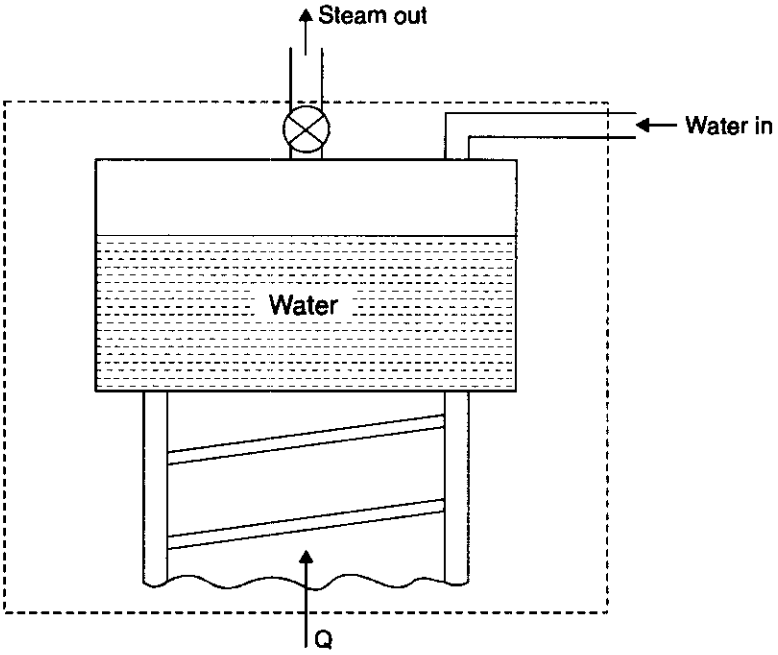


Fig. 4.37. Boiler.

For this system, $\Delta Z = 0$ and $\Delta\left(\frac{C_2^2}{2}\right) = 0$

$W = 0$ since neither any work is developed nor absorbed.

Applying energy equation to the system

$$h_1 + Q = h_2 \qquad \text{...(4.59)}$$

4.12.7. Condenser

The condenser is used to condense the steam in case of steam power plant and condense the refrigerant vapour in the refrigeration system using water or air as cooling medium. Fig. 4.38 shows the system.

For this system :

$\Delta PE = 0$, $\Delta KE = 0$ (as their values are very small compared with enthalpies)

$W = 0$ (since neither any work is developed nor absorbed)

Using energy equation to steam flow

$$h_1 - Q = h_2 \qquad \text{...[4.60 (a)]}$$

where Q = Heat lost by 1 kg of steam passing through the condenser.

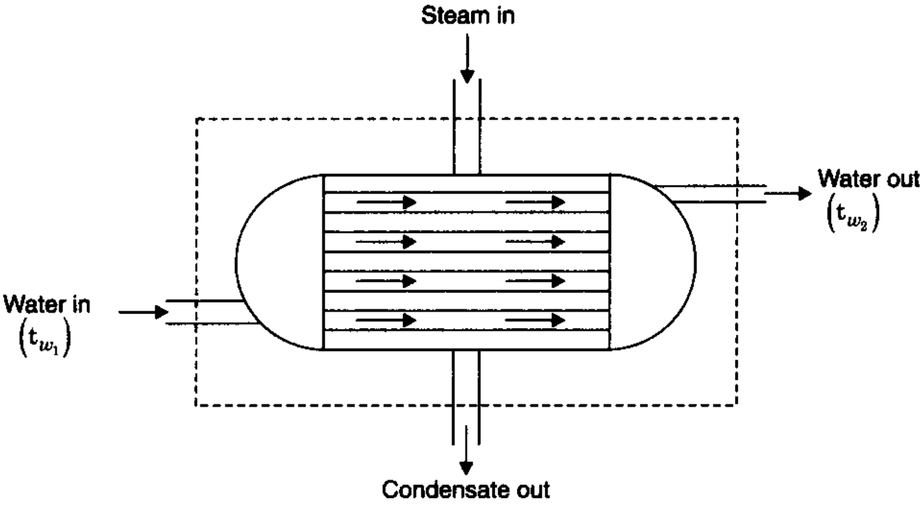


Fig. 4.38. Condenser.

Assuming there are no other heat interactions except the heat transfer between steam and water, then

$$Q = \text{Heat gained by water passing through the condenser}$$
$$= m_w (h_{w2} - h_{w1}) = m_w c_w (t_{w2} - t_{w1})$$

Substituting this value of Q in eqn. [4.60 (*a*)], we get

$$h_1 - h_2 = m_w (h_{w2} - h_{w1}) = m_w c_w (t_{w2} - t_{w1}) \qquad ...[4.60\ (b)]$$

where, m_w = Mass of cooling water passing through the condenser, and

c_w = Specific heat of water.

4.12.8. Evaporator

An evaporator is an equipment used in refrigeration plant to carry heat from the refrigerator to maintain the low temperature. Here the refrigerant liquid is passed through the evaporator and it comes out as vapour absorbing its latent heat from the surroundings of the evaporator. Fig. 4.39 shows the system. For this system

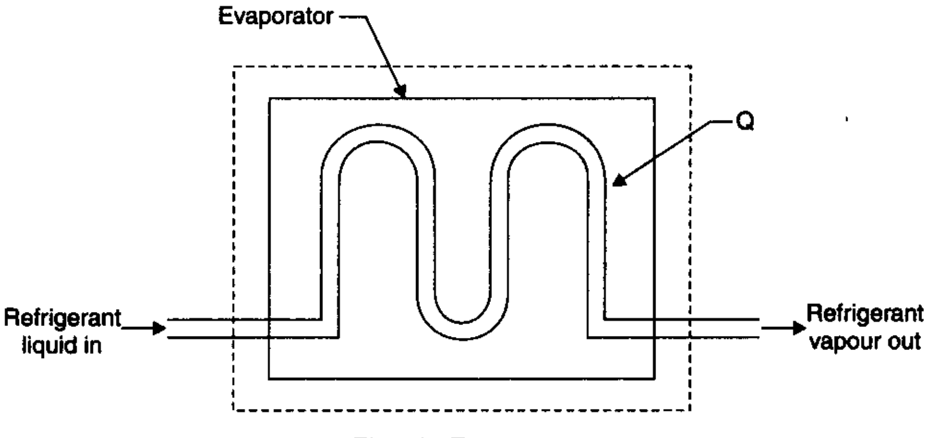


Fig. 4.39. Evaporator.

$$\Delta PE = 0,\ \Delta KE = 0$$

$$W = 0 \qquad [\because \text{ No work is absorbed or supplied}]$$

Applying the energy equation to the system

$$h_1 + Q = h_2 \qquad \text{...(4.61)}$$

Q is taken as + ve because heat *flows from the surroundings to the system* as the temperature in the system is lower than the surroundings.

4.12.9. Steam Nozzle

In case of a nozzle as the enthalpy of the fluid decreases and pressure drops simultaneously the flow of fluid is *accelerated.* This is generally used to convert the part of the energy of steam into *kinetic energy of steam* supplied to the turbine.

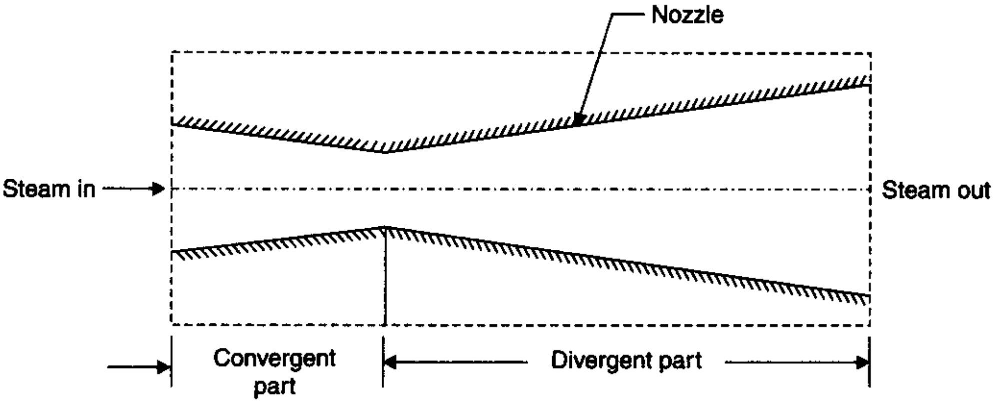


Fig. 4.40. Steam nozzle.

Fig. 4.40 shows a commonly used convergent-divergent nozzle.

For this system,

$$\Delta PE = 0$$

$$W = 0$$

$$Q = 0$$

Applying the energy equation to the system,

$$h_1 + \frac{C_1^2}{2} = h_2 + \frac{C_2^2}{2}$$

or
$$\frac{C_2^2}{2} - \frac{C_1^2}{2} = h_1 - h_2 \quad \text{or} \quad C_2^2 - C_1^2 = 2(h_1 - h_2)$$

or
$$C_2^2 = C_1^2 + 2(h_1 - h_2)$$

$$\therefore \quad C_2 = \sqrt{C_1^2 + 2(h_1 - h_2)} \qquad \text{...(4.62)}$$

where velocity C is in m/s and enthalpy h in joules.

If $C_1 << C_2$, then

$$C_2 = \sqrt{2(h_1 - h_2)} \qquad \text{...[4.63 (a)]}$$

$$\therefore \quad C_2 = \sqrt{2\Delta h}\,. \qquad \text{...[4.63 (b)]}$$

4.13. THROTTLING PROCESS AND JOULE-THOMPSON POROUS PLUG EXPERIMENT

Throttling process involves the *passage of a higher pressure fluid through a narrow constriction. The effect is the reduction in pressure and increase in volume.* This process is *adiabatic* as no heat flows from and to the system, but it is *not reversible.* It is *not an isentropic process.* The *entropy* of the fluid actually *increases.*

Such a process occurs in a flow through a *porous plug,* a *partially closed valve* and a *very narrow orifice.* The porous plug is shown in Fig. 4.41.

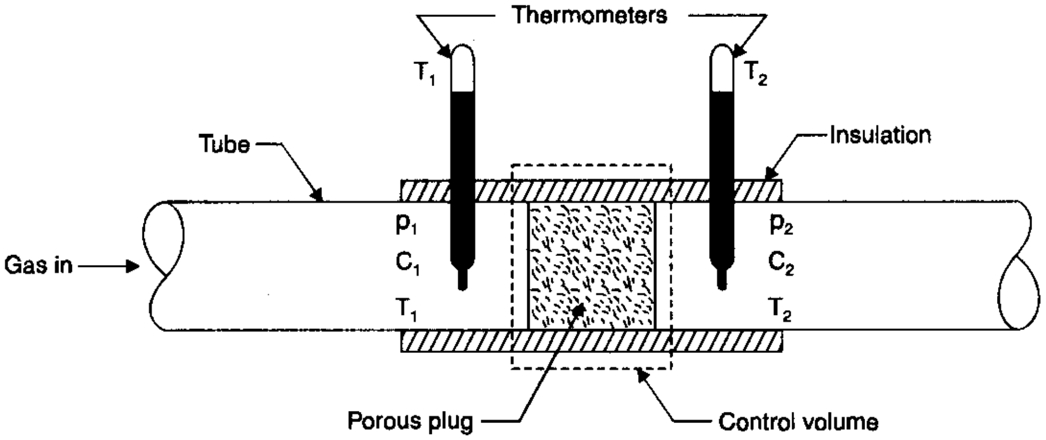


Fig. 4.41. The Joule-Thomson porous plug experiment.

In this system,

$Q = 0$ (∵ System is isolated)

$W = 0$ (∵ There is no work interaction)

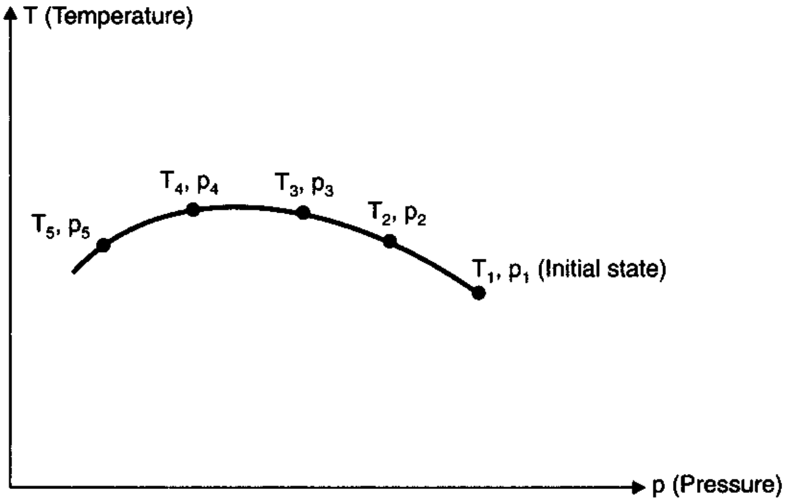


Fig. 4.42. Constant enthalpy curve.

$\Delta PE = 0$ (∵ Inlet and outlet are at the same level)

$\Delta KE = 0$ (∵ Kinetic energy does not change significantly)

Applying the energy equation to the system

$$h_1 = h_2$$

This shows that *enthalpy remains constant during adiabatic throttling process.*

The throttling process is commonly used for the following purposes :

(*i*) For determining the condition of steam (dryness fraction).

(*ii*) For controlling the speed of the turbine.

(*iii*) Used in refrigeration plant for reducing the pressure of the refrigerant before entry into the evaporator.

Throttling process frequently encountered in practice was investigated by Joule and Thompson (Lord Kelvin) in their famous *porous plug experiment* (Fig. 4.41). A stream of gas at pressure p_1 and temperature T_1 is forced continuously through a *porous plug* in a tube from which it emerges at a lower pressure p_2 and temperature T_2. The whole apparatus is *thermally insulated.*

In this process (as earlier stated)

$$h_1 = h_2$$

Whether the temperature and internal energy change in a throttling process depends on whether the fluid behaves as an ideal gas or not. Since the enthalpy of an ideal gas is a function of temperature alone, it follows that

$$T_1 = T_2 \text{ for (throttling process)}_{ideal\ gas} \quad \text{...(4.64)}$$

and, therefore, $u_1 = u_2$

For an *ideal gas,* therefore, the *throttling* process takes place at

(*i*) constant enthalpy,

(*ii*) constant temperature, and

(*iii*) constant internal energy.

The *enthalpy of a real gas is not a function of temperature alone.* In this case

$$T_1 \neq T_2 \quad \text{...(4.65)}$$

Also since the *pv* product may be different before and after throttling, the change in internal energy is not zero, as it is in *free expansion,* but is given by

$$u_2 - u_1 = p_1v_1 - p_2v_2 \quad \text{...(4.66)}$$

Joule-Thompson and Joule Co-efficients

When a real gas undergoes a throttling process a change in temperature takes place. Let us perform a series of the experiments on the same gas, keeping p_1 and T_1 constant, by varying the pressure downstream of the plug to various values p_2, p_3, p_4 etc. After throttling let T_1, T_2, T_3, T_4 etc. be the corresponding temperatures. Now if a graph is plotted between *p* and *T* (Fig. 4.42), a smooth curve drawn through these points will be a curve of *constant enthalpy* because $h_1 = h_2 = h_3 = h_4$ etc.

It may be noted that this curve does *not* represent the process executed by the gas in passing through the plug, since the process is *irreversible* and the gas does not pass through a sequence of equilibrium states.

The *slope* of a constant enthalpy line or a *p-T* diagram at a particular state may be *positive, zero* or *negative* value. The slope is called ***Joule-Thompson co-efficient,*** μ and is given by

$$\mu = \left(\frac{\partial T}{\partial p}\right)_h \quad \text{...(4.67)}$$

$$= 0 \text{ for ideal gas.}$$

If we carry out other series of experiments similar to described above starting from different initial states, we can obtain a family of constant enthalpy curves as shown in Fig. 4.43. The states where $\mu = 0$ are called '***inversion states***' and locus of these states is called the ***inversion curve***.

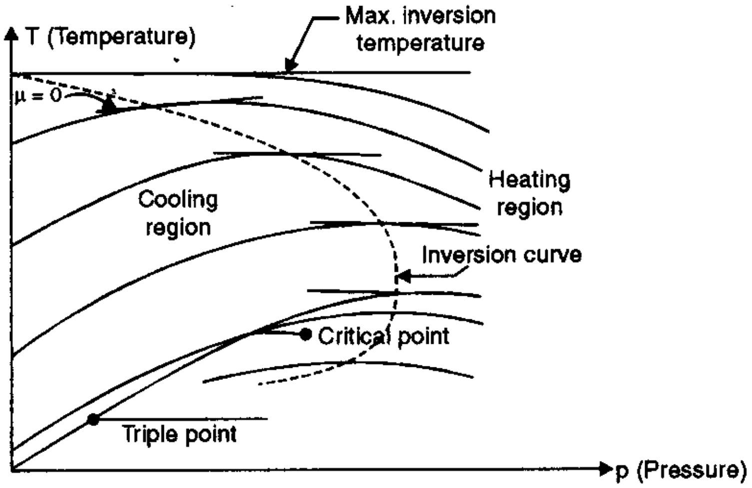


Fig. 4.43. Inversion curve.

The region inside the inversion curve is the cooling region since μ is *positive*, and temperature falls with fall in pressure.

The *region outside the inversion curve is the heating region* since μ is *negative* and temperature rises with fall in pressure.

Cooling can take place only if the initial temperature before throttling is below the *maximum inversion temperature*. This temperature is about $5T_c$.

The *maximum inversion temperatures* of some gases are given below :

(*i*) He = 24 K (*ii*) H_2 = 195 K
(*iii*) Air = 603 K (*iv*) N_2 = 261 K
(*v*) A = 732 K (*vi*) CO_2 = 1500 K.

The free expansion process is also a Joule (not Joule-Thompson). The Joule co-efficient is defined by

$$\text{Joule co-efficient} = \eta = -\left(\frac{\partial T}{\partial v}\right)_u \qquad ...(4.68)$$

For free expansion of gases the experimental data obtained is limited. From the data available it appears that η is positive (*i.e.*, cooling accompanies a fall in pressure or increase in specific volume).

Note. The throttling process is used in the liquification of gases where μ is positive.

STEADY FLOW SYSTEMS

Example 4.35. *10 kg of fluid per minute goes through a reversible steady flow process. The properties of fluid at the inlet are :* $p_1 = 1.5$ *bar,* $\rho_1 = 26$ *kg/m*^{3}*,* $C_1 = 110$ *m/s and* $u_1 = 910$ *kJ/kg and at the exit are* $p_2 = 5.5$ *bar,* $\rho_2 = 5.5$ *kg/m*^{3}*,* $C_2 = 190$ *m/s and* $u_2 = 710$ *kJ/kg. During the passage, the fluid rejects 55 kJ/s and rises through 55 metres. Determine :*

(*i*) *The change in enthalpy* (Δh) ;

(*ii*) *Work done during the process* (*W*).

Solution. Flow of fluid = 10 kg/min

Properties of fluid at the **inlet** :

Pressure, $p_1 = 1.5 \text{ bar} = 1.5 \times 10^5 \text{ N/m}^2$

Density, $\rho_1 = 26 \text{ kg/m}^3$

Velocity, $C_1 = 110 \text{ m/s}$

Internal energy, $u_1 = 910 \text{ kJ/kg}$

Properties of the fluid at the **exit** :

Pressure, $p_2 = 5.5 \text{ bar} = 5.5 \times 10^5 \text{ N/m}^2$

Density, $\rho_2 = 5.5 \text{ kg/m}^3$

Velocity, $C_2 = 190 \text{ m/s}$

Internal energy, $u_2 = 710 \text{ kJ/kg}$

Heat *rejected* by the fluid,

$$Q = 55 \text{ kJ/s}$$

Rise is elevation of fluid = 55 m.

(*i*) The change in enthalpy,

$$\Delta h = \Delta u + \Delta(pv) \quad \text{...}(i)$$

$$\Delta(pv) = \frac{p_2v_2 - p_1v_1}{1}$$

$$= \frac{p_2}{\rho_2} - \frac{p_1}{\rho_1} = \frac{5.5 \times 10^5}{5.5} - \frac{1.5 \times 10^5}{26}$$

$$= 1 \times 10^5 - 0.0577 \times 10^5$$

$$= 10^5 \times 0.9423 \text{ Nm or J} = 94.23 \text{ kJ}$$

$$\Delta u = u_2 - u_1 = (710 - 910) = -200 \text{ kJ/kg}$$

Substituting the value in eqn. (*i*), we get

$$\Delta h = -200 + 94.23 = -\mathbf{105.77 \text{ kJ/kg.}} \quad \textbf{(Ans.)}$$

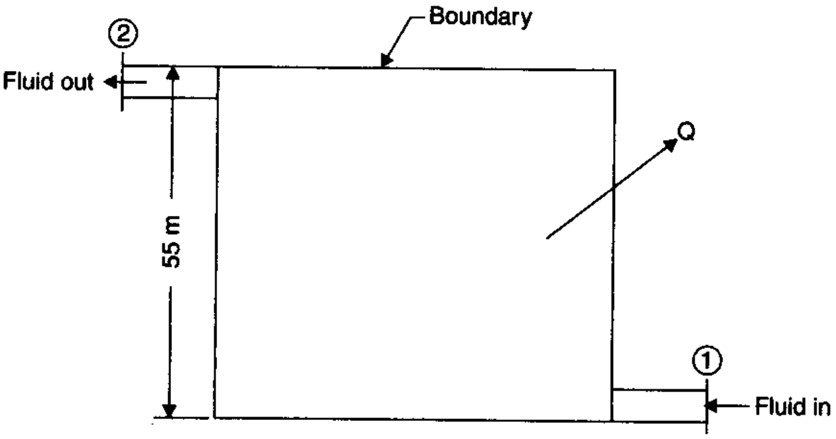


Fig. 4.44

(*ii*) The steady flow equation for unit mass flow can be written as

$$Q = \Delta KE + \Delta PE + \Delta h + W$$

where Q is the heat transfer per kg of fluid

$$Q = 55 \text{ kJ/s} = \frac{55 \text{ kJ/s}}{\frac{10}{60} \text{ kg/s}} = 55 \times 6 = 330 \text{ kJ/kg}$$

Now, $$\Delta KE = \frac{C_2^2 - C_1^2}{2} = \frac{(190)^2 - (110)^2}{2} \text{ Nm or J} = 12000 \text{ J or } 12 \text{ kJ/kg}$$

$$\Delta PE = (Z_2 - Z_1)\, g = (55 - 0) \times 9.81 \text{ Nm or J} = 539.5 \text{ J or} \approx 0.54 \text{ kJ/kg}$$

Substituting the value in steady flow equation,

$$-330 = 12 + 0.54 - 105.77 + W \quad \text{or} \quad W = -236.77 \text{ kJ/kg}.$$

Work done per second $= -236.77 \times \frac{10}{60} = -39.46$ kJ/s $= -$ **39.46 kW. (Ans.)**

Example 4.36. *In a gas turbine unit, the gases flow through the turbine is 15 kg/s and the power developed by the turbine is 12000 kW. The enthalpies of gases at the inlet and outlet are 1260 kJ/kg and 400 kJ/kg respectively, and the velocity of gases at the inlet and outlet are 50 m/s and 110 m/s respectively. Calculate :*

(i) The rate at which heat is rejected to the turbine, and

(ii) The area of the inlet pipe given that the specific volume of the gases at the inlet is 0.45 m³/kg.

Solution. Rate of flow of gases, $\dot{m} = 15$ kg/s

Volume of gases at the inlet, $v = 0.45$ m³/kg

Power developed by the turbine, $P = 12000$ kW

∴ Work done, $W = \frac{12000}{15} = 800$ kJ/kg

Enthalpy of gases at the inlet, $h_1 = 1260$ kJ/kg

Enthalpy of gases at the oulet, $h_2 = 400$ kJ/kg

Velocity of gases at the inlet, $C_1 = 50$ m/s

Velocity of gases at the outlet, $C_2 = 110$ m/s.

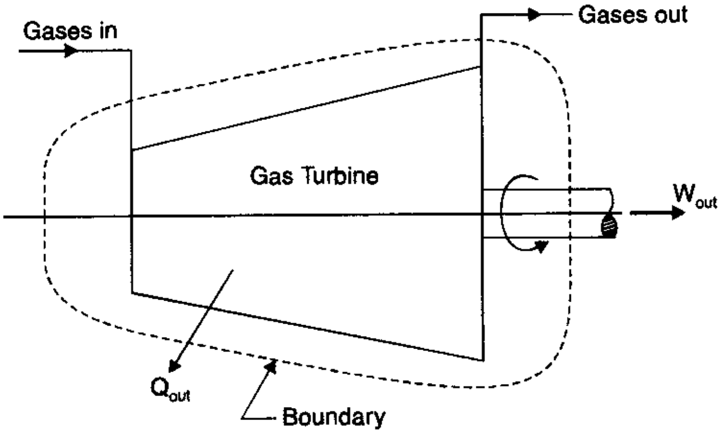


Fig. 4.45

(*i*) **Heat rejected, Q :**

Using the flow equation,

$$h_1 + \frac{C_1^2}{2} + Q = h_2 + \frac{C_2^2}{2} + W \qquad ...(i) \; [\because \; Z_1 = Z_2]$$

Kinetic energy at inlet $= \frac{C_1^2}{2} = \frac{50^2}{2}\ \text{m}^2/\text{s}^2 = \frac{50^2\ \text{kg m}^3}{2\ \text{s}^2\text{kg}} = 1250\ \text{Nm/kg} = 1.25\ \text{kJ/kg}$

Kinetic energy at outlet $= \frac{C_1^2}{2} = \frac{110^2}{2 \times 1000} = 6.05\ \text{kJ/kg}$

Substituting these values in eqn. (*i*), we get

$$1260 + 1.25 + Q = 400 + 6.05 + 800$$

$$\therefore \qquad Q = -\ 55.2\ \text{kJ/kg}$$

i.e., **Heat rejected** $= +\ 55.2\ \text{kJ/kg} = 55.2 \times 15\ \text{kJ/s} =$ **828 kW. (Ans.)**

(*ii*) **Inlet area, A :**

Using the relation,

$$\dot{m} = \frac{CA}{v}$$

$$\therefore \qquad A = \frac{v\dot{m}}{C} = \frac{0.45 \times 15}{50} = \mathbf{0.135\ m^2}.\ \textbf{(Ans.)}$$

☞**Example 4.37.** *In an air compressor air flows steadily at the rate of 0.5 kg/s through an air compressor. It enters the compressor at 6 m/s with a pressure of 1 bar and a specific volume of 0.85 m³/kg and leaves at 5 m/s with a pressure of 7 bar and a specific volume of 0.16 m³/kg. The internal energy of the air leaving is 90 kJ/kg greater than that of the air entering. Cooling water in a jacket surrounding the cylinder absorbs heat from the air at the rate of 60 kJ/s. Calculate :*

(*i*) *The power required to drive the compressor ;*

(*ii*) *The inlet and output pipe cross-sectional areas.*

Solution. Air flow rate through the compressor, $\dot{m} = 0.5$ kg/s

Velocity of air at the inlet to compressor, $C_1 = 6$ m/s

Velocity of air at the outlet of compressor, $C_2 = 5$ m/s

Pressure of air at the inlet to the compressor, $p_1 = 1$ bar

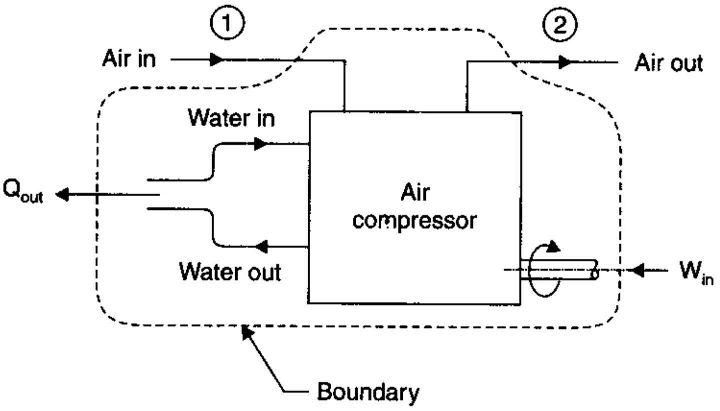


Fig. 4.46

Pressure of air at outlet to the compressor, $p_2 = 7$ bar

Specific volume of air at inlet to the compressor, $v_1 = 0.85$ m^3/kg

Specific volume of air at outlet to the compressor, $v_2 = 0.16$ m^3/kg

Difference of internal energy at the outlet and inlet of the compressor,

$$(u_2 - u_1) = 90 \text{ kJ/kg}$$

Heat rejected by air (to cooling water),

$$Q = -\frac{60}{0.5} = -120 \text{ kJ/kg.}$$

(*i*) **Power required to drive the compressor :**

Using the steady flow energy equation,

$$u_1 + \frac{C_1^2}{2} + p_1v_1 + Q = u_2 + \frac{C_2^2}{2} + p_2v_2 + W$$

$$\therefore \qquad W = (u_1 - u_2) + \left(\frac{C_1^2}{2} - \frac{C_2^2}{2}\right) + (p_1v_1 - p_2v_2) + Q$$

$$= -90 + \frac{1}{1000}\left(\frac{6^2}{2} - \frac{5^2}{2}\right) + \frac{10^5}{1000}(1 \times 0.85 - 7 \times 0.16) + (-120)$$

$$= -90 + 0.0055 - 27 - 120 = -237 \text{ kJ/kg (app.).}$$

(Note that the change in kinetic energy is negligibly small in comparison with the other terms).

i.e., Work input required = 237 kJ/kg = 237 × 0.5 kJ/s = 118.5 kW

Hence, **power required to drive the compressor = 118.5 kW. (Ans.)**

(*ii*) **Inlet and outlet pipe cross-sectional areas, A_1 and A_2 :**

Using the relation,

$$\dot{m} = \frac{CA}{v}$$

$$\therefore \qquad A_1 = \frac{\dot{m}v_1}{C_1} = \frac{0.5 \times 0.85}{6} \text{m}^2 = 0.0708 \text{ m}^2$$

i.e., Inlet pipe cross-sectional area, A_1 = **0.0708 m². (Ans.)**

Again, $$A_2 = \frac{\dot{m}v_2}{C_2} = \frac{0.5 \times 0.16}{5} \text{m}^2 = 0.016 \text{ m}^2.$$

i.e., Outlet pipe cross-sectional area, A_2 = **0.016 m². (Ans.)**

Note. In this example, the steady flow energy equation has been used, despite the fact the compression consists of : suction of air ; compression in a closed cylinder ; and discharge of air. The steady flow equation can be used because the cycle of processes takes place many times in a minute, and therefore, average effect is steady flow of air through the machine.

Example 4.38. *In a steam plant, 1 kg of water per second is supplied to the boiler. The enthalpy and velocity of water entering the boiler are 800 kJ/kg and 5 m/s. The water receives 2200 kJ/kg of heat in the boiler at constant pressure. The steam after passing through the turbine comes out with a velocity of 50 m/s, and its enthalpy is 2520 kJ/kg. The inlet is 4 m above the turbine exit. Assuming the heat losses from the boiler and the turbine to the surroundings are*

20 kJ/s, calculate the power developed by the turbine. Consider the boiler and turbine as single system.

Solution. Enthalpy of water entering the boiler, $h_1 = 800$ kJ/kg

Velocity of water entering the boiler, $C_1 = 5$ m/s

Enthalpy of steam at the outlet of the turbine, $h_2 = 2520$ kJ/kg

Velocity of steam at the outlet of the turbine, $C_2 = 50$ m/s

Elevation difference, $(Z_1 - Z_2) = 4$ m

Net heat added to the water in the boiler, $Q = 2200 - 20 = 2180$ kJ/kg

Power developed by the turbine :

Using the flow equation,

$$h_1 + \frac{C_1^2}{2} + Z_1g + Q = h_2 + \frac{C_2^2}{2} + Z_2g + W$$

$$\therefore \quad W = (h_1 - h_2) + \left(\frac{C_1^2}{2} - \frac{C_2^2}{2}\right) + (Z_1 - Z_2)\,g + Q$$

$$= (800 - 2520) + \frac{1}{1000}\left[\frac{5^2}{2} - \frac{50^2}{2}\right] + \frac{4 \times 9.81}{1000} + 2180$$

$$= -\,1720 + \frac{1}{1000}(12.5 - 1250) + \frac{39.24}{1000} + 2180$$

$$= -\,1720 - 1.2375 + 0.03924 + 2180$$

$$= 458.8 \text{ kJ/kg} = 458.8 \text{ kJ/s} = 458.8 \text{ kW}$$

Hence, **power developed by the turbine = 458.8 kW. (Ans.)**

Example 4.39. *A turbine, operating under steady-flow conditions, receives 4500 kg of steam per hour. The steam enters the turbine at a velocity of 2800 m/min, an elevation of 5.5 m and a specific enthalpy of 2800 kJ/kg. It leaves the turbine at a velocity of 5600 m/min, an elevation of 1.5 m and a specific enthalpy of 2300 kJ/kg. Heat losses from the turbine to the surroundings amout to 16000 kJ/h.*

Determine the power output of the turbine.

Solution. Quantity of steam supplied to the turbine, $m = 4500$ kg/h

Steam velocity at the entrance to the turbine, $C_1 = 2800$ m/min

Elevation at the entrance, $Z_1 = 5.5$ m

Specific enthalpy at the entrance, $h_1 = 2800$ kJ/g

Steam velocity at the exit, $C_2 = 5600$ m/min

Elevation at the exit, $Z_2 = 1.5$ m

Specific enthalpy at the exit, $h_2 = 2300$ kJ/kg

Heat losses from the turbine to the surroundings, $Q = -\,16000$ kJ/h

Applying the steady flow energy equation at entry (1) and exit (2)

$$m\left(h_1 + \frac{C_1^2}{2} + Z_1g\right) + Q = m\left(h_2 + \frac{C_2^2}{2} + Z_2g\right) + W$$

$$\therefore \quad Q - W = m\left[(h_2 - h_1) + \left(\frac{C_2^2 - C_2^2}{2}\right) + (Z_2 - Z_1)g\right]$$

$$-\frac{16000}{3600} - W = \frac{4500}{3600}\left[(2300 - 2800) + \left\{\frac{\left(\frac{5600}{60}\right)^2 - \left(\frac{2800}{60}\right)^2}{2 \times 1000}\right\} + \frac{(1.5 - 5.5) \times 9.81}{1000}\right]$$

$-4.44 - W = 1.25\ (500 + 3.26 - 0.039)$ or $W = 633.44$ kJ/s

$\therefore$ **Power output of the turbine = 633.44 kW. (Ans.)**

Example 4.40. *Steam at a 6.87 bar, 205°C, enters in an insulated nozzle with a velocity of 50 m/s. It leaves at a pressure of 1.37 bar and a velocity of 500 m/s.*

Determine the final enthalpy of steam.

Solution. Pressure of steam at the entrance, $p_1 = 6.87$ bar

The velocity with which steam enters the nozzle, $C_1 = 50$ m/s

Pressure of steam at the exit, $p_2 = 1.37$ bar

Velocity of steam at the exit, $C_2 = 500$ m/s.

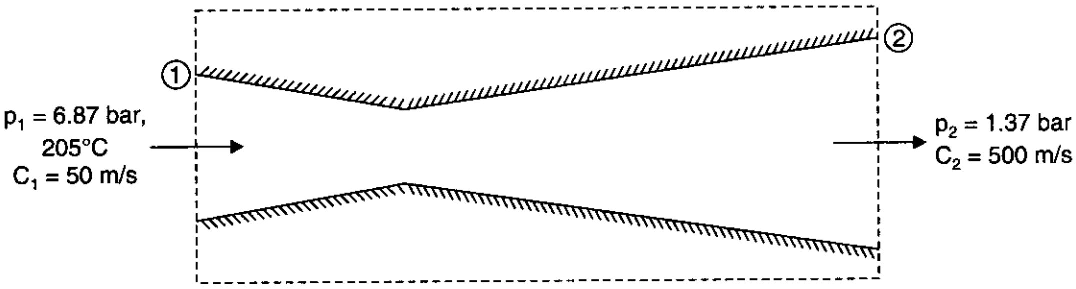


Fig. 4.47

The steady flow energy equation is given by

$$h_1 + \frac{C_1^2}{2} + Z_1g + Q = h_2 + \frac{C_2^2}{2} + Z_2g + W \qquad ...(i)$$

Considering the nozzle as an open system, it is evident that :

— there is *no work transfer* across the boundary of the system (*i.e.*, $W = 0$)

— there is *no heat transfer* because the nozzle is insulated (*i.e.*, $Q = 0$).

— the change in potential energy is negligible since there is no significant difference in elevation between the entrance and exit of the nozzle [*i.e.* $(Z_2 - Z_1)\,g = 0$].

Thus eqn. (*i*) reduces to

$$h_1 + \frac{C_1^2}{2} = h_2 + \frac{C_2^2}{2}$$

$$\therefore \quad (h_2 - h_1) + \frac{C_2^2 - C_1^2}{2} = 0$$

From steam table corresponding to 6.87 bar, $h_1 = 2850$ kJ/kg

$$\therefore \quad (h_2 - 2850) + \frac{(500)^2 - (50)^2}{2 \times 1000} = 0$$

or $h_2 - 2850 + 123.75 = 0$ or $h_2 = 2726.25$ kJ

Hence final enthalpy of steam = 2726.25 kJ. (Ans.)

Example 4.41. *The working fluid, in a steady flow process flows at a rate of 220 kg/min. The fluid rejects 100 kJ/s passing through the system. The conditions of the fluid at inlet and outlet are given as :* $C_1 = 320$ *m/s,* $p_1 = 6.0$ *bar,* $u_1 = 2000$ *kJ/kg,* $v_1 = 0.36$ *m³/kg and* C_2 = *140 m/s,* $p_2 = 1.2$ *bar,* $u_2 = 1400$ *kJ/kg,* $v_2 = 1.3$ *m³/kg. The suffix 1 indicates the condition at inlt and 2 indicates at outlet of the system.*

Determine the power capacity of the system in MW.

The change in potential energy may be neglected.

Solution. Refer Fig. 4.48.

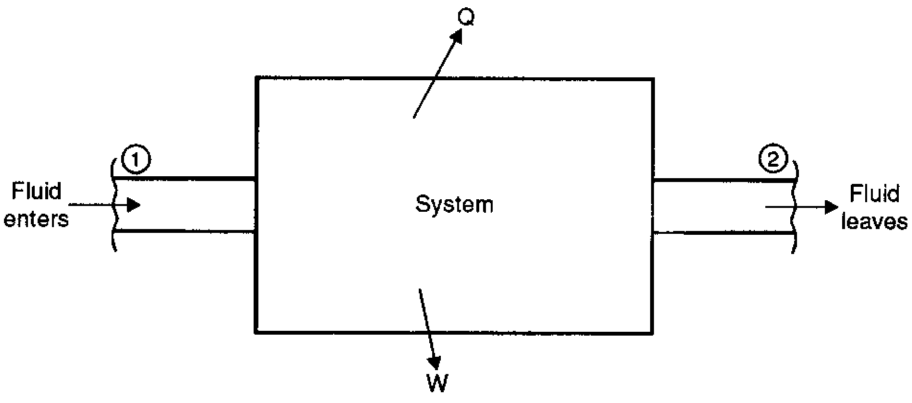


Fig. 4.48

Conditions of the fluid at point 1 :

Velocity, $C_1 = 320$ m/s
Pressure, $p_1 = 6.0$ bar $= 6 \times 10^5$ N/m²
Internal energy, $u_1 = 2000$ kJ/kg
Specific volume, $v_1 = 0.36$ m³/kg.

Conditions of the fluid at point 2 :

Velocity, $C_2 = 140$ m/s
Pressure, $p_2 = 1.2$ bar $= 1.2 \times 10^5$ N/m²
Internal energy, $u_2 = 1400$ kJ/kg
Specific volume, $v_2 = 1.3$ m³/kg
Heat rejected by the fluid, $Q = 100$ kJ/s (–).

Power capacity of the system :

Applying the energy equation at '1' and '2', we get

$$m\left[u_1 + p_1v_1 + \frac{C_1^2}{2} + Z_1g\right] \pm Q = m\left[u_1 + p_2v_2 + \frac{C_2^2}{2} + Z_2g\right] \pm W$$

Taking –ve sign for Q as the system *rejects* heat and +ve sign for W as the system *develops work.*

$$m\left[u_1 + p_1v_1 + \frac{C_1^2}{2} + Z_1g\right] - Q = m\left[u_2 + p_2v_2 + \frac{C_2^2}{2} + Z_2g\right] + W$$

$$\therefore \quad W = m\left[(u_1 - u_2) + (p_1v_1 - p_2v_2) + \left(\frac{C_1^2 - C_2^2}{2}\right)\right] - Q\,. \qquad [\because \ (Z_1 - Z_2)\,g = 0]$$

In the above equation :

— the mass flow is in kg/s

— velocity in m/s

— internal energy in J/kg

— pressure in N/m^2

— specific volume m^3/kg

— the value of Q is in J/s

Then the unit of W will be J/s.

$$\therefore \quad W = \frac{220}{60}\left[(2000 - 1400) \times 10^3 + 10^5\,(6 \times 0.36 - 1.2 \times 1.3) + \left(\frac{320^2 - 140^2}{2}\right)\right] - 100 \times 10^3$$

$$= \frac{220}{60}\,[600 \times 10^3 + 10^5 \times 0.6 + 41.4 \times 10^3] - 100 \times 10^3$$

$$= \frac{220}{60}\,[600 \times 10^3 + 60 \times 10^3 + 41.4 \times 10^3] - 100 \times 10^3$$

$$= 10^3 \times 2471.8 \text{ J/s} \qquad [\because \; 1 \text{ kJ} = 10^3 \text{ J}]$$

$$= 2471.8 \text{ kJ/s or kW} = 2.4718 \text{ MW}$$

Hence **power capacity of the system = 2.4718 MW. (Ans.)**

☞**Example 4.42.** *A stream of gases at 7.5 bar, 750°C and 140 m/s is passed through a turbine of a jet engine. The stream comes out of the turbine at 2.0 bar, 550°C and 280 m/s. The process may be assumed adiabatic. The enthalpies of gas at the entry and exit of the turbine are 950 kJ/kg and 650 kJ/kg of gas respectively.*

Determine the capacity of the turbine if the gas flow is 5 kg/s.

Solution. Refer Fig. 4.49.

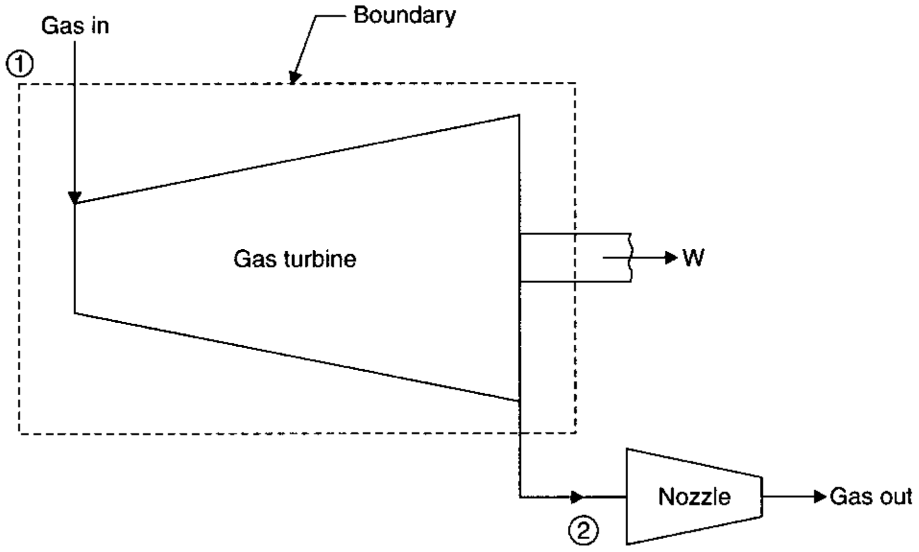


Fig. 4.49

Conditions at '1' :

Pressure, $p_1 = 7.5 \text{ bar} = 7.5 \times 10^5 \text{ N/m}^2$, 750°C

Velocity, $C_1 = 140 \text{ m/s}$

Enthalpy, $h_1 = 950 \text{ kJ/kg}$

Conditions at '2' :

Pressure, $p_2 = 2.0 \text{ bar} = 2 \times 10^5 \text{ N/m}^2$, 550°C

Velocity, $C_2 = 280 \text{ m/s}$

Enthalpy, $h_2 = 650 \text{ kJ/kg}$

Gas flow, $m = 5 \text{ kg/s}$.

Capacity of the turbine :

Considering the flow of gas as 1 kg and neglecting the change in potential energy, we can write the steady flow energy equation for the turbine as

$$h_1 + \frac{C_1^2}{2} \pm Q = h_2 + \frac{C_2^2}{2} \pm W$$

$Q = 0$ as the system is adiabatic and W should be taken as +ve since it develops work.

$$\therefore \quad h_1 + \frac{C_1^2}{2} = h_2 + \frac{C_2^2}{2} + W$$

$$\therefore \quad W = (h_1 - k_2) + \frac{C_1^2 - C_2^2}{2} = 10^3 (950 - 650) + \frac{140^2 - 280^2}{2}$$

$$= 10^3 \times 300 - 29.4 \times 10^3$$

$$= 270.6 \times 10^3 \text{ J/kg} = 270.6 \text{ kJ/kg}.$$

Power capacity of the turbine

$$= \dot{m}W = 5 \times 270.6 = 1353 \text{ kJ/s}$$

$$= \mathbf{1353 \text{ kW}}. \quad \textbf{(Ans.)}$$

☞**Example 4.43.** *12 kg of air per minute is delivered by a centrifugal air compressor. The inlet and outlet conditions of air are $C_1 = 12$ m/s, $p_1 = 1$ bar, $v_1 = 0.5$ m³/kg and $C_2 = 90$ m/s, $p_2 = 8$ bar, $v_2 = 0.14$ m³/kg. The increase in enthalpy of air passing through the compressor is 150 kJ/kg and heat loss to the surroundings is 700 kJ/min.*

Find : (i) Motor power required to drive the compressor ;

(ii) Ratio of inlet to outlet pipe diameter.

Assume that inlet and discharge lines are at the same level.

Solution. Quantity of air delivered by the compressor, $m = \frac{12}{60} = 0.2$ kg/s.

Conditions of air at the inlet 1 :

Velocity, $C_1 = 12 \text{ m/s}$

Pressure, $p_1 = 1 \text{ bar} = 1 \times 10^5 \text{ N/m}^2$

Specific volume, $v_1 = 0.5 \text{ m}^3\text{/kg}$

Conditions of air at the outlet 2 :

Velocity, $C_2 = 90 \text{ m/s}$

Pressure, $p_2 = 8 \text{ bar} = 8 \times 10^5 \text{ N/m}^2$

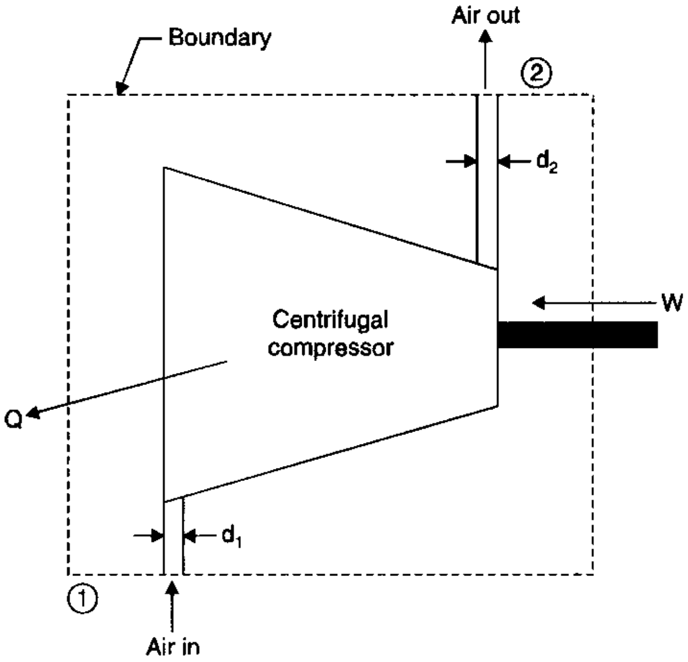


Fig. 4.50

Specific volume, $v_2 = 0.14\ \text{m}^3/\text{kg}$

Increase in enthalpy of air passing through the compressor,

$$(h_2 - h_1) = 150\ \text{kJ/kg}$$

Heat lost to the surroundings,

$$Q = -\ 700\ \text{kJ/min} = -\ 11.67\ \text{kJ/s}.$$

(*i*) **Motor power required to drive the compressor :**

Applying energy equation to the system,

$$m\left(h_1 + \frac{C_1^2}{2} + Z_1 g\right) + Q = m\left(h_2 + \frac{C_2^2}{2} + Z_2 g\right) + W$$

Now $Z_1 = Z_2$ (given)

$$\therefore \quad m\left(h_1 + \frac{C_1^2}{2}\right) + Q = m\left(h_2 + \frac{C_2^2}{2}\right) + W$$

$$W = m\left[(h_1 - h_2) + \frac{C_1^2 - C_2^2}{2}\right] + Q$$

$$= 0.2\left[-\ 150 + \frac{12^2 - 90^2}{2 \times 1000}\right] + (-\ 11.67)$$

$$= -\ 42.46\ \text{kJ/s} = -\ 42.46\ \text{kW}$$

∴ **Motor power required (or work done on the air) = 42.46 kW. (Ans.)**

(*ii*) **Ratio of inlet to outlet pipe diameter,** $\mathbf{\frac{d_1}{d_2}}$ **:**

The mass of air passing through the compressor is given by

$$m = \frac{A_1C_1}{v_1} = \frac{A_2C_2}{v_2}$$

$$\therefore \qquad \frac{A_1}{A_2} = \frac{C_2}{C_1} \times \frac{v_1}{v_2} = \frac{90}{12} \times \frac{0.5}{0.14} = 26.78$$

$$\therefore \qquad \left(\frac{d_1}{d_2}\right)^2 = 26.78 \quad \text{or} \quad \frac{d_1}{d_2} = 5.175$$

Hence ratio of inlet to outlet pipe diameter = 5.175. (Ans.)

Example 4.44. *In a test of water cooled air compressor, it is found that the shaft work required to drive the compressor is 175 kJ/kg of air delivered and the enthalpy of air leaving is 70 kJ/kg greater than that entering and that the increase in enthalpy of circulating water is 92 kJ/kg.*

Compute the amount of heat transfer to the atmosphere from the compressor per kg of air.

Solution. Refer Fig. 4.51.

Shaft work required to drive the compressor, $W = -175$ kJ/kg

Increase in enthalpy of air passing through the compressor, $(h_2 - h_1) = 70$ kJ/kg

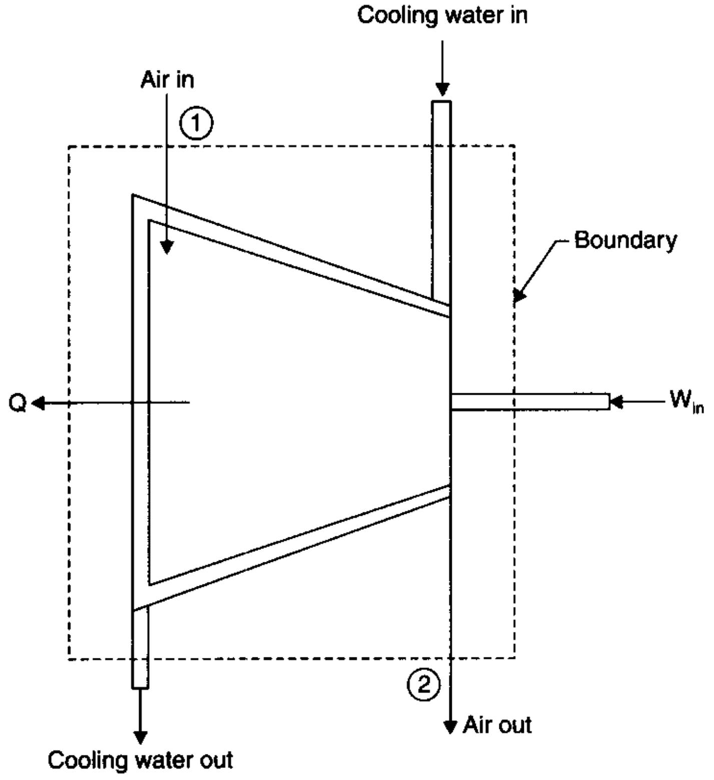


Fig. 4.51

Increase in enthalpy of circulating water, $Q_{water} = -92$ kJ/kg

Amount of heat transferred to atmosphere, $Q_{atm.} = ?$

Applying steady-flow energy equation at '1' and '2', we get

$$h_1 + \frac{C_1^2}{2} + Z_1 g + Q = h_2 + \frac{C_2^2}{2} + Z_2 g + W$$

or $$Q = (h_2 - h_1) + \left(\frac{C_2^2 - C_1^2}{2}\right) + (Z_2 - Z_1)g + W$$

Assuming change in P.E. and K.E. to be negligible.

$\therefore \quad Q = (h_2 - h_1) + W = 70 + (-175) = -105$ kJ

But $\quad Q = Q_{atm} + Q_{water}$ or $-105 = Q_{atm} + (-92)$

$\therefore \quad Q_{atm} = -13$ kJ/kg.

Thus heat transferred to atmosphere = 13 kJ/kg. (Ans.)

☞**Example 4.45.** *At the inlet to a certain nozzle the enthalpy of fluid passing is 2800 kJ/kg, and the velocity is 50 m/s. At the discharge end the enthalpy is 2600 kJ/kg. The nozzle is horizontal and there is negligible heat loss from it.*

(i) Find the velocity at exit of the nozzle.

(ii) If the inlet area is 900 cm² and the specific volume at inlet is 0.187 m³/kg, find the mass flow rate.

(iii) If the specific volume at the nozzle exit is 0.498 m³/kg, find the exit area of nozzle.

Solution. Refer Fig. 4.52.

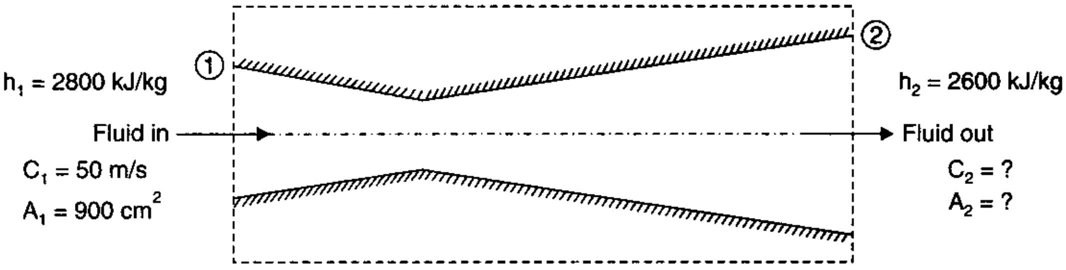


Fig. 4.52

Conditions of fluid at inlet (1) :

Enthalpy, $h_1 = 2800$ kJ/kg

Velocity, $C_1 = 50$ m/s

Area, $A_1 = 900 \text{ cm}^2 = 900 \times 10^{-4} \text{ m}^2$

Specific volume, $v_1 = 0.187 \text{ m}^3/\text{kg}$

Conditions of fluid at exit (2) :

Enthalpy, $h_2 = 2600$ kJ/kg

Specific volume, $v_2 = 0.498 \text{ m}^3/\text{kJ}$

Area, $A_2 = ?$

Mass flow rate, $\dot{m} = ?$

(*i*) **Velocity at exit of the nozzle, C_2 :**

Applying energy equation at '1' and '2', we get

$$h_1 + \frac{C_1^2}{2} + Z_1g + Q = h_2 + \frac{C_2^2}{2} + Z_2g + W$$

Here $Q = 0,\ W = 0,\ Z_1 = Z_2$

$$\therefore \quad h_1 + \frac{C_1^2}{2} = h_2 + \frac{C_2^2}{2}$$

$$\frac{C_2^2}{2} = (h_1 - h_2) + \frac{C_1^2}{2}$$

$$= (2800 - 2600) \times 1000 + \frac{50^2}{2} = 201250 \text{ N-m}$$

$$\therefore \quad C_2^2 = 402500$$

$$\therefore \quad \mathbf{C_2 = 634.4 \text{ m/s.}} \quad \textbf{(Ans.)}$$

(*ii*) **Mass flow rate $\dot{m}$:**

By continuity equation,

$$\dot{m} = \frac{AC}{v} = \frac{A_1C_1}{v_1} = \frac{900 \times 10^{-4} \times 50}{0.187} \text{ kg/s} = 24.06 \text{ kg/s}$$

∴ **Mass flow rate** **= 24.06 kg/s.** **(Ans.)**

(*iii*) **Area at the exit, A_2 :**

Now, $$\dot{m} = \frac{A_2\,C_2}{v_2}$$

$$24.06 = \frac{A_2 \times 634.4}{0.498}$$

$$\therefore \quad A_2 = \frac{24.06 \times 0.498}{634.4} = 0.018887 \text{ m}^2 = 188.87 \text{ cm}^2$$

Hence, **area at the exit** **= 188.87 cm^2.** **(Ans.)**

Example 4.46. *In one of the sections of the heating plant in which there are no pumps enters a steady flow of water at a temperature of 50°C and a pressure of 3 bar (h = 240 kJ/kg). The water leaves the section at a temperature of 35°C and at a pressure of 2.5 bar (h = 192 kJ/kg). The exit pipe is 20 m above the entry pipe.*

Assuming change in kinetic energy to be negligible, evaluate the heat transfer from the water per kg of water flowing.

Solution. Refer Fig. 4.53.

Enthalpy at '1', $h_1 = 240$ kJ/kg

Enthalpy at '2', $h_2 = 192$ kJ/kg

$Z_2 - Z_1 = 20$ m

Applying steady flow energy equation,

$$h_1 + \frac{C_1^2}{2} + Z_1g + Q = h_2 + \frac{C_2^2}{2} + Z_2g + W$$

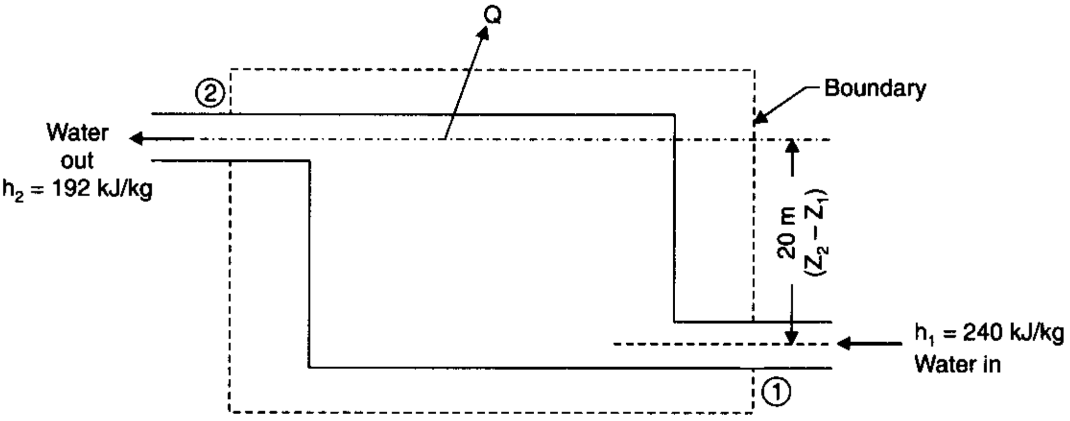


Fig. 4.53

$$Q = (h_2 - h_1) + \left(\frac{C_2^2 - C_1^2}{2}\right) + (Z_2 - Z_1)g + W$$

Here $W = 0$ (no pumps)

$$\frac{C_2^2 - C_1^2}{2} = 0 \qquad \text{(given)}$$

$$\therefore \qquad Q = (192 - 240) + \frac{20 \times 9.81}{1000} = -47.8 \text{ kJ/kg}$$

∴ **Heat transfer from water/kg = 47.8 kJ/kg. (Ans.)**

Example 4.47. *The gas leaving the turbine of a gas turbine jet engine flows steadily into the engine jet pipe at a temperature of 900°C, a pressure of 2 bar and a velocity of 300 m/s relative to the pipe. Gas leaves the pipe at a temperature of 820°C and a pressure of 1.1 bar. Heat transfer from the gas is negligible. Using the following data evaluate the relative velocity of gas leaving the jet pipe. For the gas at t = 820°C, h = 800 kJ/kg and at 910°C, 915 kJ/kg.*

Solution. Pressure at entry to the engine jet pipe, $p_1 = 2$ bar

Velocity relative to the pipe, $C_1 = 300$ m/s

Heat transfer from gas, $Q = 0$

At temperature, $t_1 = 910°C$, $h_1 = 915$ kJ/kg

At temperature, $t_2 = 820°C$, $h_2 = 800$ kJ/kg

Relative velocity of gas leaving the jet pipe, C_2 :

Steady flow energy equation is given by :

$$h_1 + \frac{C_1^2}{2} + Z_1 g + Q = h_2 + \frac{C_2^2}{2} + Z_2 g + W$$

$$Q = 0$$

$$W = 0$$

$$Z_1 = Z_2 \text{ (assumed)}$$

$$h_1 + \frac{C_1^2}{2} = h_2 + \frac{C_2^2}{2}$$

$$\frac{C_2^2}{2} = (h_1 - h_2) + \frac{C_1^2}{2} = (915 - 800) \times 1000 + \frac{300^2}{2}$$

$$\therefore \qquad C_2^2 = 320000 \quad \text{or} \quad C_2 = 565.7 \text{ m/s.}$$

Hence relative velocity of gas leaving the jet pipe = 565.7 m/s. (Ans.)

Example 4.48. *A centrifugal pump delivers 50 kg of water per second. The inlet and outlet pressures are 1 bar and 4.2 bar respectively. The suction is 2.2 m below the centre of the pump and delivery is 8.5 m above the centre of the pump. The suction and delivery pipe diameters are 20 cm and 10 cm respectively.*

Determine the capacity of the electric motor to run the pump.

Solution. Refer Fig. 4.54.

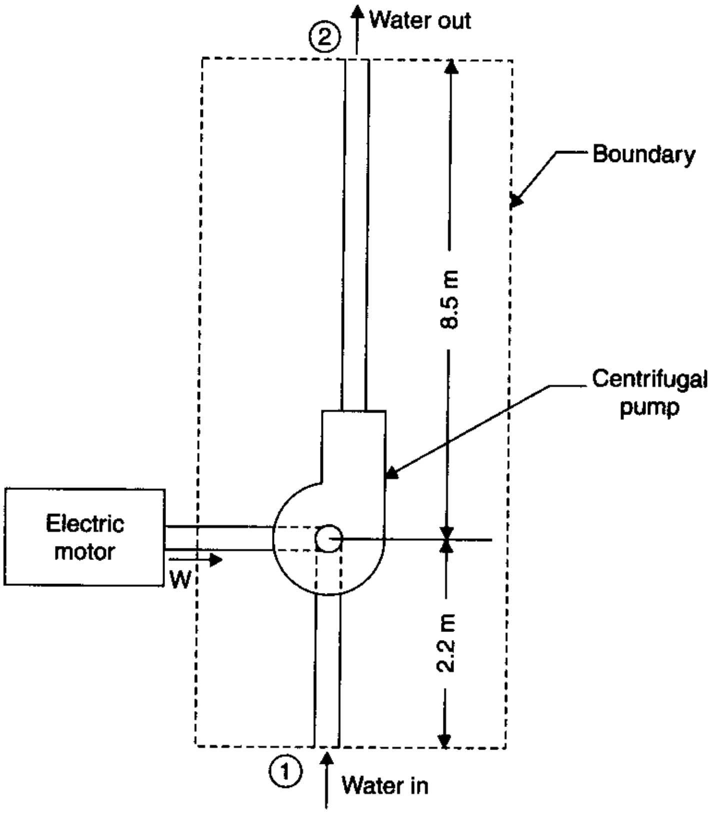


Fig. 4.54

Quantity of water delivered by the pump, m_w = 50 kg/s

Inlet pressure, p_1 = 1 bar = 1×10^5 N/m^2

Outlet pressure, p_2 = 4.2 bar = 4.2×10^5 N/m^2

Suction-below the centre of the pump = 2.2 m

Delivery-above the centre of the pump = 8.5 m

Diameter of suction pipe, d_1 = 20 cm = 0.2 m

Diameter of delivery pipe, d_2 = 10 cm = 0.1 m

Capacity of electric motor :

Steady flow energy equation is given by

$$m_w\left(u_1 + p_1v_1 + \frac{C_1^2}{2} + Z_1g\right) + Q = m_w\left(u_2 + p_2v_2 + \frac{C_2^2}{2} + Z_2g\right) + W \quad ...(i)$$

Considering the datum from suction 1, as shown

$Z_1 = 0,\ Z_2 = 8.5 + 2.2 = 10.7$ m

$u_2 - u_1 = 0\ ;\ Q = 0$

Thus eqn. (*i*) reduces to

$$W = m_w\left[(p_1v_1 - p_2v_2) + (Z_1 - Z_2)g + \left(\frac{C_1^2 - C_2^2}{2}\right)\right] \quad ...(ii)$$

As water is incompressible fluid.

$$\therefore \qquad v_2 = v_1 = v = \frac{1}{\rho} = \frac{1}{1000}$$

The mass flow through inlet and exit pipe is given by

$$m_w = \frac{\pi}{4} \times d_1^2 \times C_1 \times \rho = \frac{\pi}{4} \times d_2^2 \times C_2 \times \rho \text{ as } \rho_1 = \rho_2 = \rho \text{ (for water)}$$

$$\therefore \qquad 50 = \frac{\pi}{4} \times (0.2)^2 \times C_1 \times 1000$$

$$\therefore \qquad C_1 = \frac{50 \times 4}{\pi \times (0.2)^2 \times 1000} = 1.59 \text{ m/s}$$

and

$$C_2 = \frac{50 \times 4}{\pi \times (0.1)^2 \times 1000} = 6.37 \text{ m/s}$$

Substituting the values in eqn. (*ii*), we get

$$W = 50\left[\left(1 \times 10^5 \times \frac{1}{1000} - 4.2 \times 10^5 \times \frac{1}{1000}\right) + (0 - 10.7) \times 9.81 + \left(\frac{1.59^2 - 6.37^2}{2}\right)\right]$$

$$= 50[-320 - 104.96 - 19.02]$$

$$= 22199 \text{ J/s or } 22.199 \text{ kJ/s} \simeq 22.2 \text{ kW.}$$

Hence capacity of electric motor = **22.2 kW. (Ans.)**

Example 4.49. *During flight, the air speed of a turbojet engine is 250 m/s. Ambient air temperature is – 14°C. Gas temperature at outlet of nozzle is 610°C. Corresponding enthalpy values for air and gas are respectively 250 and 900 kJ/kg. Fuel air ratio is 0.0180. Chemical energy of fuel is 45 MJ/kg. Owing to incomplete combustion 6% of chemical energy is not released in the reaction. Heat loss from the engine is 21 kJ/kg of air.*

Calculate the velocity of the exhaust jet.

Solution. Refer Fig. 4.55.

Air speed of turbojet engine, $C_a = 250$ m/s

Ambient air temperature = – 14°C

Gas temperature at outlet of nozzle = 610°C

Enthalpy of air, $h_a = 250$ kJ/kg

Enthalpy of gas, $h_g = 900$ kJ/kg

Fuel air ratio $= 0.0180$

$$\left[\begin{array}{l}\text{If, mass of air, } m_a = 1 \text{ kg, then mass of fuel, } m_f = 0.018 \text{ kg} \\ \text{and mass of gas} = 1 + .018 = 1.018 \text{ kg}\end{array}\right]$$

Chemical energy of the fuel = 45 MJ/kg.

Heat loss from the engine, Q = 21 kJ/kg of air

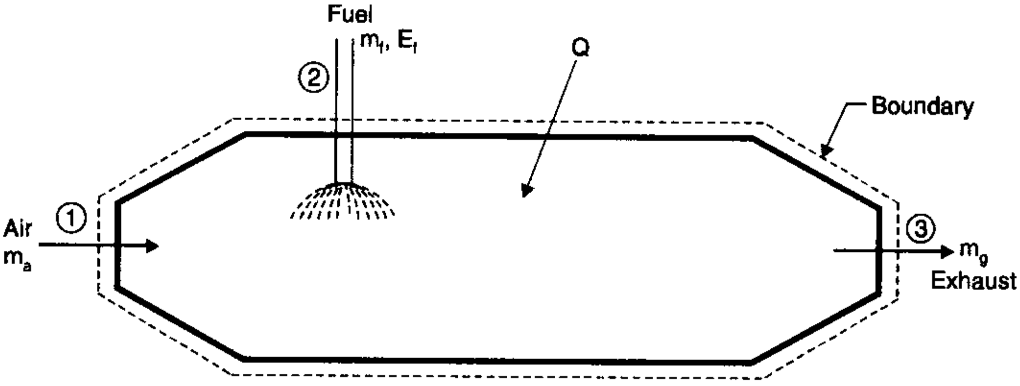


Fig. 4.55

Velocity of the exhaust gas jet, C_g :

Energy equation for turbojet engine is given by,

$$m_a\left(h_a + \frac{C_a^2}{2}\right) + m_f E_f + Q = m_g\left(h_g + \frac{C_g^2}{2} + E_g\right)$$

$$1\left(250 + \frac{250^2}{2 \times 1000}\right) + 0.018 \times 45 \times 10^3 + (-21)$$

$$= 1.018\left[900 + \frac{C_g^2}{2 \times 1000} + 0.06 \times \frac{0.018}{1.018} \times 45 \times 10^3\right]$$

$$281.25 + 810 - 21 = 1.018\left(900 + \frac{C_g^2}{2000} + 47.74\right)$$

$$1070.25 = 1.018\left(947.74 + \frac{C_g^2}{2000}\right)$$

$$\therefore \quad C_g = 455.16 \text{ m/s}$$

Hence, **velocity of exhaust gas jet = 455.16 m/s. (Ans.)**

☞**Example 4.50.** *Air at a temperature of 20°C passes through a heat exchanger at a velocity of 40 m/s where its temperature is raised to 820°C. It then enters a turbine with same velocity of 40 m/s and expands till the temperature falls to 620°C. On leaving the turbine, the air is taken at a velocity of 55 m/s to a nozzle where it expands until the temperature has fallen to 510°C. If the air flow rate is 2.5 kg/s, calculate :*

(*i*) *Rate of heat transfer to the air in the heat exchanger ;*

(*ii*) *The power output from the turbine assuming no heat loss ;*

(iii) The velocity at exit from the nozzle, assuming no heat loss.

Take the enthalpy of air as $h = c_p t$, where c_p is the specific heat equal to 1.005 kJ/kg°C and t the temperature.

Solution. Refer Fig. 4.56.

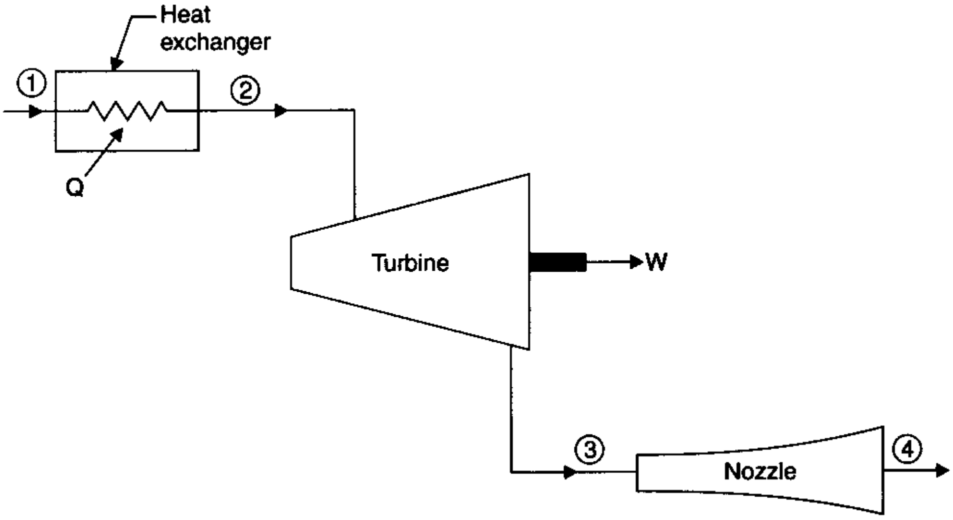


Fig. 4.56

Temperature of air, $t_1 = 20°C$

Velocity of air, $C_1 = 40$ m/s.

Temperature of air after passing the heat exchanger, $t_2 = 820°C$

Velocity of air at entry to the turbine, $C_2 = 40$ m/s

Temperature of air after leaving the turbine, $t_3 = 620°C$

Velocity of air at entry to nozzle, $C_3 = 55$ m/s

Temperature of air after expansion through the nozzle, $t_4 = 510°C$

Air flow rate, $\dot{m} = 2.5$ kg/s.

(*i*) **Heat exchanger :**

Rate of heat transfer :

Energy equation is given as,

$$m\left(h_1 + \frac{C_1^2}{2} + Z_1 g\right) + Q_{1-2} = m\left(h_2 + \frac{C_2^2}{2} + Z_2 g\right) + W_{1-2}$$

Here, $Z_1 = Z_2,\ C_1,\ C_2 = 0,\ W_{1-2} = 0$

$\therefore$ $mh_1 + Q_{1-2} = mh_2$

or $Q_{1-2} = m(h_2 - h_1)$

$= mc_p(t_2 - t_1) = 2.5 \times 1.005\ (820 - 20) = 2010$ kJ/s.

Hence, **rate of heat transfer = 2010 kJ/s. (Ans.)**

(*ii*) **Turbine :**

Power output of turbine :

Energy equation for turbine gives

$$m\left(h_2+\frac{C_2^{\,2}}{2}\right)=m\left(h_3+\frac{C_3^{\,2}}{2}\right)+W_{2\text{-}3} \qquad [\because \; Q_{2\text{-}3}=0,\; Z_1=Z_2]$$

$$\therefore \quad W_{2-3}=m\left(h_2+\frac{C_2^{\,2}}{2}\right)-m\left(h_3+\frac{C_3^{\,2}}{2}\right)$$

$$=m\left[(h_2-h_3)+\left(\frac{C_2^{\,2}-C_3^{\,2}}{2}\right)\right]$$

$$=m\left[c_p\,(t_2-t_3)+\frac{C_2^{\,2}-C_3^{\,2}}{2}\right]$$

$$=2.5\left[1.005\,(820-620)+\frac{(40)^2-(55)^2}{2\times1000}\right]$$

$$=2.5\;[201+0.7125]=504.3\text{ kJ/s or }504.3\text{ kW}$$

Hence, **power output of turbine = 504.3 kW. (Ans.)**

(*iii*) **Nozzle :**

Velocity at exit from the nozzle :

Energy equation for nozzle gives,

$$h_3+\frac{C_3^{\,2}}{2}=h_4+\frac{C_4^{\,2}}{2} \qquad [\because \; W_{3\text{-}4}=0,\; Q_{3-4}=0,\; Z_1=Z_2]$$

$$\frac{C_4^{\,2}}{2}=(h_3-h_4)+\frac{C_3^{\,2}}{2}=c_p(t_3-t_4)+\frac{C_3^{\,2}}{2}$$

$$=1.005(620-510)+\frac{55^2}{2\times1000}=112.062\times10^3\text{ J}$$

$$\therefore \quad C_4=473.4\text{ m/s.}$$

Hence, **velocity at exit from the nozzle = 473.4 m/s. (Ans.)**

4.14. HEATING-COOLING AND EXPANSION OF VAPOURS

The basic energy equations for non-flow and flow processes are *also valid for vapours.*

∴ When $\Delta KE = 0$ and $\Delta PE = 0$

$dQ = du + p.dv$ ......for *non-flow process.*

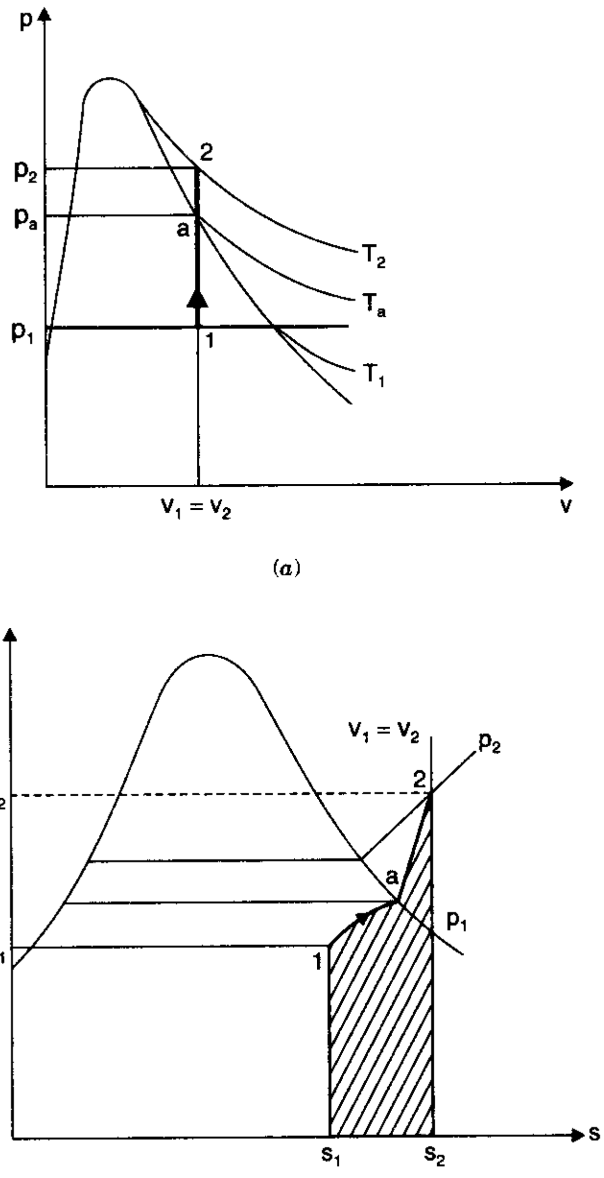
$dQ = dh - v.dp$ ......for *flow process.*

The various processes using vapour are discussed below :

1. **Constant Volume Heating or Cooling.** The constant volume heating process is represented on *p-v*, *T-s* and *h-s* diagram as shown in Fig. 4.57 (*a*), (*b*), (*c*) respectively. It is assumed that the steam is in wet condition before heating at pressure p_1, becomes superheated after heating and pressure increases from p_1 to p_2.

Since the mass of steam, *m*, remains constant during the heating process,

$$\therefore \quad m=\frac{V}{x_1 v_{g_1}}=\frac{V}{v_{sup_2}}$$, where *V* is the total constant volume of steam



(a)

(b)

Also $$\frac{v_{sup_2}}{v_{g_2}} = \frac{T_{sup_2}}{T_{s_2}}$$

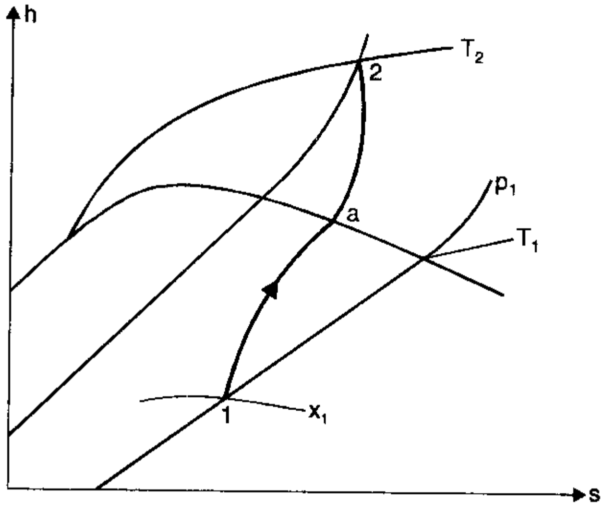
$$\therefore \qquad \frac{T_{sup_2}}{T_{s_2}} = \frac{x_1 v_{g_1}}{v_{g_2}} \qquad \text{...(4.69)}$$

v_{g_2} and T_{s_2} can be found from the steam tables corresponding to pressure p_2 and then T_{sup_2} can be calculated by using the above equation. When the final condition is known, the change in all other properties can be found easily.

If after cooling the condition of steam remains wet, then the mass fraction be obtained as follows :

$$\frac{V}{x_1 v_{g_1}} = \frac{V}{x_2 v_{g_2}}$$

$$x_2 = \frac{x_1 v_{g_1}}{v_{g_2}} \qquad \text{...(4.70)}$$



(c)

Fig. 4.57. Constant volume process.

where v_{g_2} can be found from the steam tables corresponding to pressure p_2.

Applying the first law of thermodynamics, we have

$$Q = \Delta u + \int_1^2 p \, . \, dv = \Delta u \text{ as } \int_1^2 p dv = 0$$

$$= u_2 - u_1$$

i.e.,

$$Q_1 = [h_2 - p_2 v_{sup_2}] - [h_1 - p_1(x_1 v_{g_1})] \qquad \text{...(4.71)}$$

In case the condition of steam remains wet after heating, then

$$Q = (u_2 - u_1) = [h_2 - p_2(x_2 v_{g_2})] - [h_1 - p_1(x_1 v_{g_1})] \qquad \text{...(4.72)}$$

In the *cooling process,* the same equations are used except that the suffixes 1, 2 are interchanged.

☞**Example 4.51.** *A rigid cylinder of volume 0.028 m³ contains steam at 80 bar and 350°C. The cylinder is cooled until the pressure is 50 bar. Calculate :*

(*i*) *The state of steam after cooling ;*

(*ii*) *The amount of heat rejected by the steam.*

Solution. Volume of rigid cylinder = 0.028 m³

Pressure of steam before cooling, p_1 = 80 bar

Temperature of steam before cooling = 350°C

Pressure of steam after cooling, p_2 = 50 bar

Steam at 80 bar and 350°C is in a superheated state, and the specific volume from tables is 0.02995 m^3/kg. Hence the mass of steam in the cylinder is given by

$$m = \frac{0.028}{0.02995} = 0.935 \text{ kg}$$

Internal energy at state 1, (*80 bar, 350°C*),

$$u_1 = h_1 - p_1 v_1$$

$$= 2987.3 - \frac{80 \times 10^5 \times 0.02995}{10^3} \quad \text{or} \quad u_1 = 2747.7 \text{ kJ/kg.}$$

(*i*) **State of steam after cooling :**

At state 2, p_2 = 50 bar and v_2 = 0.02995 m^3/kg, therefore, steam is wet, and dryness fraction is given by,

$$x_2 = \frac{v_2}{v_{g_2}} = \frac{0.02995}{0.0394} = 0.76.$$

(*ii*) **Heat rejected by the steam :**

Internal energy at state 2 (50 bar),

$$u_2 = (1 - x_2)\, u_{f_2} + x_2 u_{g_2}$$

$$= (1 - 0.76) \times 1149 + 0.76 \times 2597 = 2249.48 \text{ kJ/kg}$$

At constant volume, $Q = U_2 - U_1 = m(u_2 - u_1)$

$$= 0.935(2249.48 - 2747.7) = -\, 465.5 \text{ kJ}$$

i.e., **Heat rejected** = **465.5 kJ. (Ans.)**

Fig. 4.58 shows the process drawn on *T-s* diagram, the shaded area representing the heat rejected by the system.

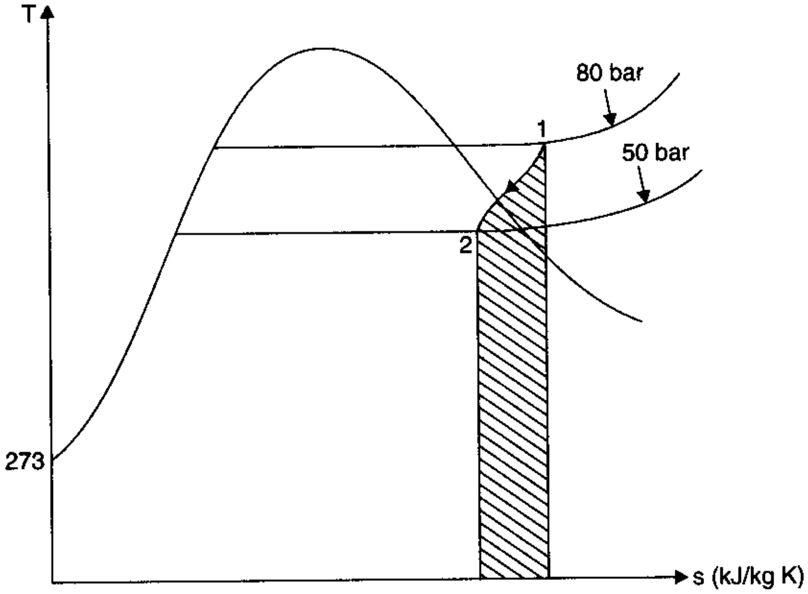


Fig. 4.58

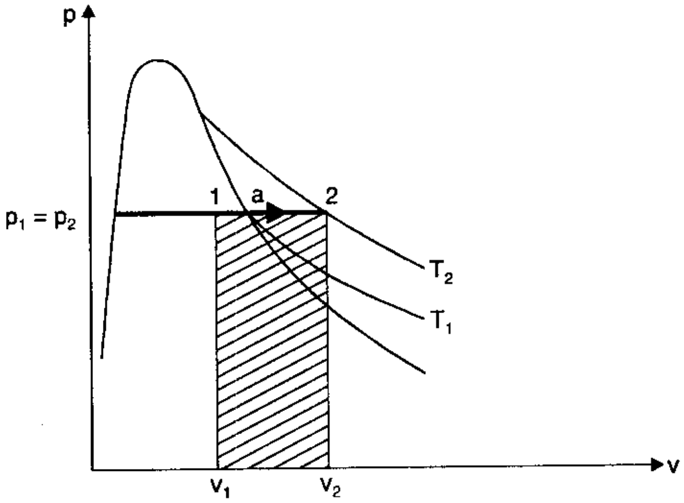
2. **Constant pressure Heating or Cooling.** Fig. 4.59 (*a*), (*b*) and (*c*) shows the constant pressure heating process on *p-v*, *T-s* and *h-s* diagrams respectively.

Generation of steam in the boilers is an example of constant pressure heating.

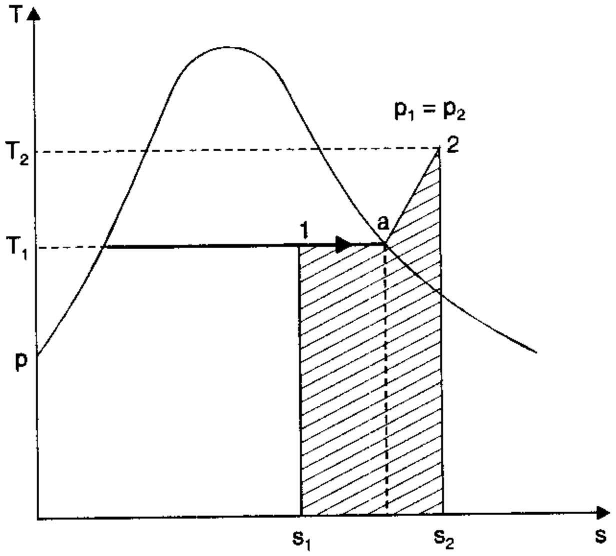
Applying first law of thermodynamics, we have

$$Q = \Delta u + \int_1^2 p \, . \, dv$$

$$= (u_2 - u_1) + p(v_2 - v_1) = (u_2 + pv_2) - (u_1 + pv_1) = h_2 - h_1$$



(*a*)



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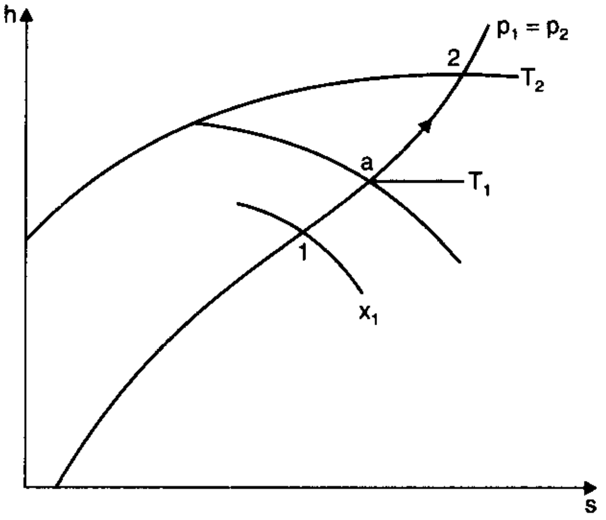


(c)

Fig. 4.59. Constant pressure process.

If the initial condition of steam is wet and final condition is superheated, then

$$Q = (u_2 + pv_{sup_2}) - (u_1 + p \,.\, x_1 \, v_{g_1})$$

$$= (h_2 - h_1) \qquad ...(4.73)$$

here h_1 and h_2 are the actual enthalpies of steam per kg before and after heating.

The heat added during the constant pressure process is equal to the change in enthalpy of steam during the process. When the steam is wet before heating and becomes superheated after heating the work done,

$$W = p\,(v_{sup_2} - x_1 v_{g_1}) \qquad ...(4.74)$$

Example 4.52. *0.08 kg of dry steam is heated at a constant pressure of 2 bar until the volume occupied is 0.10528 m³. Calculate :*

(*i*) *Heat supplied ;*

(*ii*) *Work done.*

Solution. Mass of steam, $m = 0.08$ kg

Pressure of steam, $p = 2$ bar

Volume occupied after heating = 0.10528 m³

Initially the steam is dry saturated at 2 bar, hence

$$h_1 = h_g \text{ (at 2 bar)} = 2706.3 \text{ kJ/kg}$$

Finally the steam is at 2 bar and the specific volume is given by

$$v_2 = \frac{0.10528}{0.08} = 1.316 \text{ m}^3\text{/kg}$$

Hence the steam is *superheated finally* (since the value of v_g at 2 bar = 0.885 m³/kg). From superheat tables at 2 bar and 1.316 m³/kg the temperature of steam is 300°C, and the enthalpy, h_2 = 3071.8 kJ/kg.

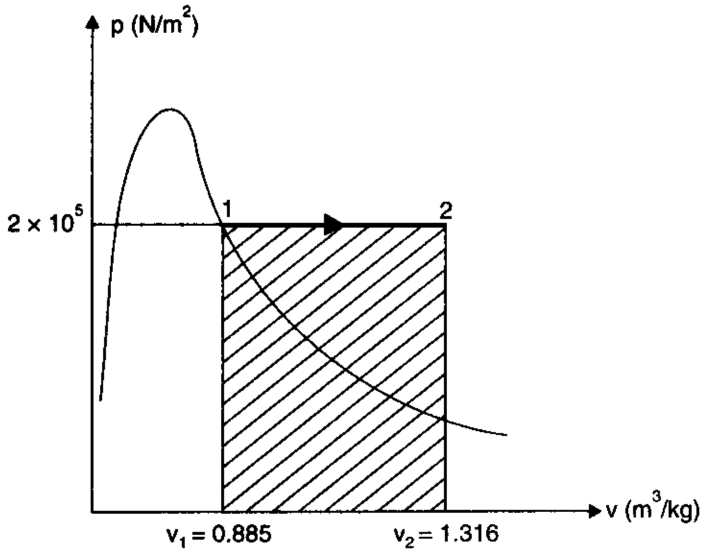


Fig. 4.60

(*i*) **Heat supplied :**

Heat supplied, $Q = H_2 - H_1 = m(h_2 - h_1)$

$= 0.08(3071.8 - 2706.3)$

$=$ **29.24 kJ. (Ans.)**

(*ii*) **Work done :**

The process is shown on a *p-v* diagram in Fig. 4.60. The work done is given by the shaded area

i.e., $W = p(v_2 - v_1)$ Nm/kg

Here $v_1 = v_g$ at 2 bar $= 0.885$ m^3/kg

and $v_2 = 1.316$ m^3/kg

$\therefore$ $W = 2 \times 10^5\,(1.316 - 0.885) = 2 \times 10^5 \times 0.431$ Nm/kg

Now work done by the total mass of steam (0.08 kg) present

$= 0.08 \times 2 \times 10^5 \times 0.431 \times 10^{-3}$ kJ

$=$ **6.896 kJ. (Ans.)**

Example 4.53. *1 kg of steam at 8 bar, entropy 6.55 kJ/kg K, is heated reversibly at constant pressure until the temperature is 200°C. Calculate the heat supplied, and show on a T-s diagram the area which represents the heat flow.*

Solution. Mass of steam, $m = 1$ kg

Pressure of steam, $p = 8$ bar

Entropy of steam (at 8 bar), $s = 6.55$ kJ/kg K

Temperature after heating = 200°C

At 8 bar, $s_g = 6.66$ kJ/kg K, hence steam is *wet*, since the actual entropy, s, is less than s_g.

To find the dryness fraction x_1, using the relation,

$$s_1 = s_{f_1} + x_1 s_{fg_1}$$

$$6.55 = 2.0457 + x_1 \times 4.6139$$

$$\therefore \quad x_1 = \frac{6.55 - 2.0457}{4.6139} = 0.976$$

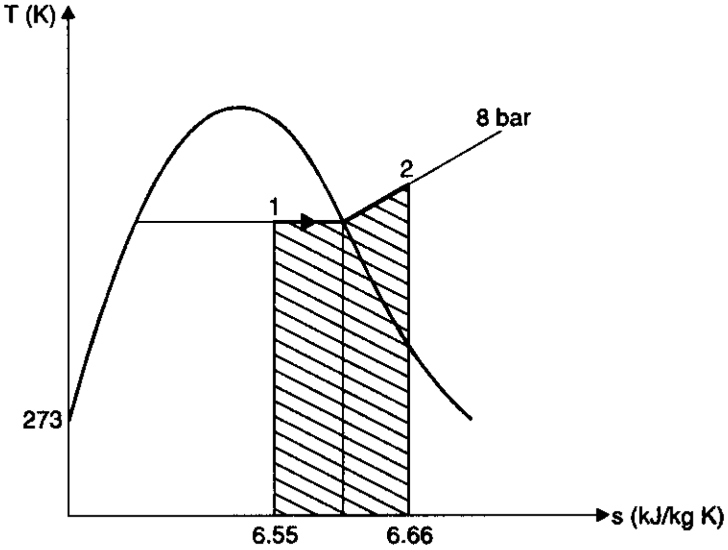


Fig. 4.61

Now, *initial enthalpy* (at 8 bar),

$$h_1 = h_{f_1} + x_1 h_{fg_1}$$

$$= 720.9 + 0.976 \times 2046.5 = 2718.28 \text{ kJ/kg}$$

Final enthalpy, h_2 : At state 2 the steam is at 200°C at 8 bar and is therefore, superheated.

From superheated tables, $h_2 = 2839.3$ kJ/kg

Now, $Q = h_2 - h_1 = 2839.3 - 2718.28 = 121.02$ kJ/kg

i.e., **Heat supplied = 121.02 kJ/kg. (Ans.)**

The *T-s* diagram showing the process is given in Fig. 4.61, the shaded area representing the heat flow.

3. Constant Temperature or Isothermal Expansion. Fig. 4.62 (*a*), (*b*) and (*c*) shows the constant temperature or isothermal expansion on *p-v*, *T-s* and *h-s* diagrams respectively.

In the wet region, the constant temperature process is also a constant pressure process during evaporation and as well as condensation. When the steam becomes saturated it behaves like a gas and constant temperature process in superheated region becomes hyperbolic (*pv* = constant).

When the wet steam is heated at constant temperature till it becomes *dry and saturated,* then the heat transfer (*Q*) is given by :

$$Q = h_2 - h_1$$

and work done,

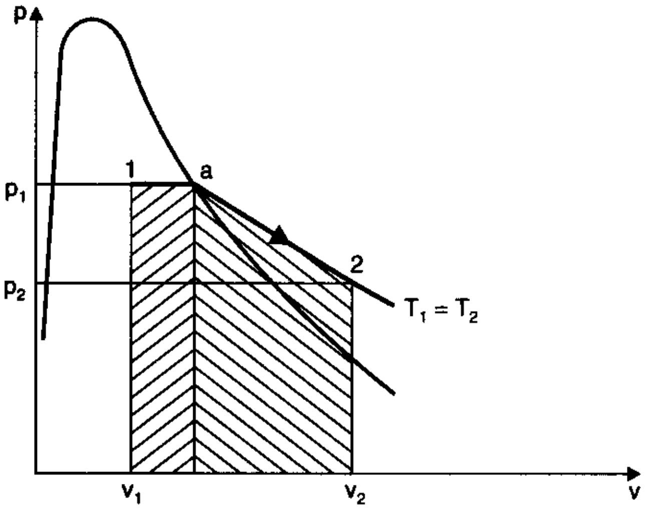
$$W = p_1(v_{g_2} - x_1 v_{g_1})$$

$$= pv_{g_1}(1 - x_1)$$

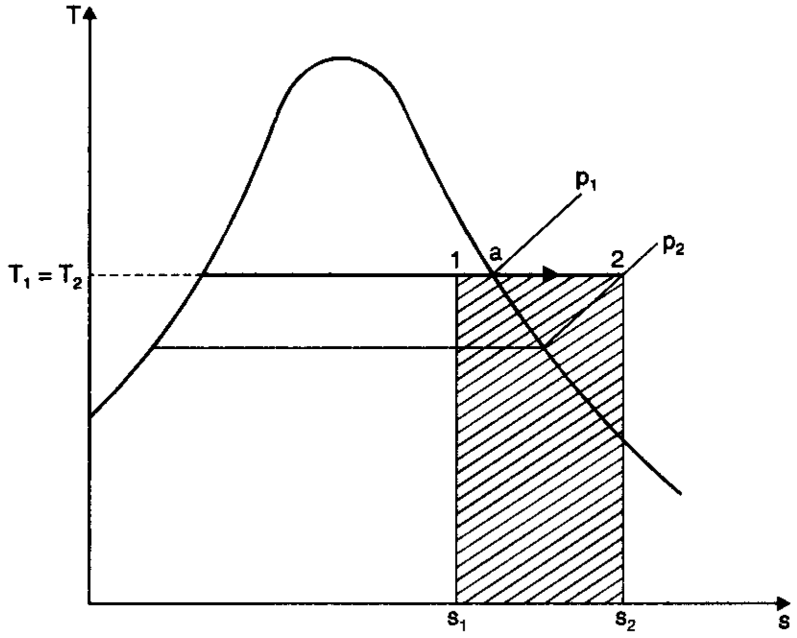
$[\because \; v_{g_2} = v_{g_1}$ as pressure remains constant during this process]

This process is limited to *wet region only.*

Hyperbolic process (pv = constant) is also an isothermal process in the superheat region as the steam behaves like a gas in this region. The work done during the hyperbolic expansion in a *non-flow system* is given by



(*a*)



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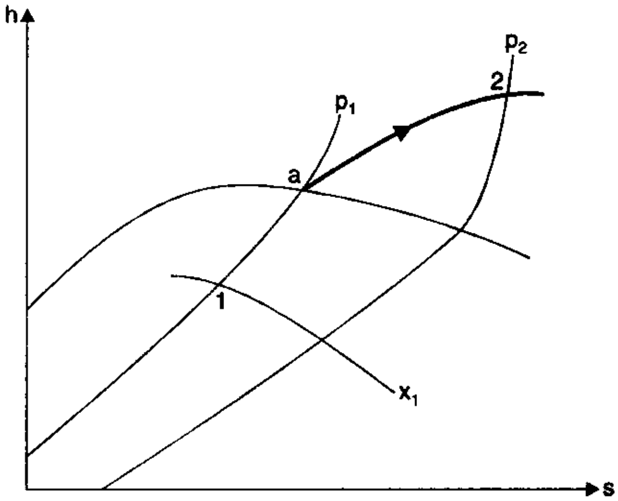


(c)

Fig. 4.62. Constant temperature or isothermal expansion.

$$W = \int_1^2 p dv = \int_1^2 \frac{C}{v} dv = C \log_e \left(\frac{v_2}{v_1}\right)$$

$$= p_1 v_1 \log_e \left(\frac{v_2}{v_1}\right) \qquad ...(4.75)$$

where v_1 and v_2 are the specific volumes of steam before and after expansion.

Applying first law of energy equation,

$$Q = \Delta u + \int_1^2 p \, . \, dv$$

$$= (u_2 - u_1) + p_1 v_1 \log_e \left(\frac{v_2}{v_1}\right)$$

$$= (h_2 - p_2 v_2) - (h_1 - p_1 v_1) + p_1 v_1 \log_e \left(\frac{v_2}{v_1}\right)$$

Since $p_1 v_1 = p_2 v_2$

$$\therefore \quad Q = (h_2 - h_1) + p_1 v_1 \log_e \frac{v_2}{v_1} \qquad ...(4.76)$$

☞**Example 4.54.** *Steam at 7 bar and dryness fraction 0.95 expands in a cylinder behind a piston isothermally and reversibly to a pressure of 1.5 bar. The heat supplied during the process is found to be 420 kJ/kg. Calculate per kg :*

(i) The change of internal energy ; *(ii) The change of enthalpy ;*

(iii) The work done.

Solution. Initial pressure of steam, $p_1 = 7$ bar $= 7 \times 10^5$ N/m^2

Final pressure of steam, $p_2 = 1.5$ bar $= 1.5 \times 10^5$ N/m^2

Heat supplied during the process, $Q = 420$ kJ/kg.

The process is shown in Fig. 4.63. The saturation temperature corresponding to 7 bar is 165°C. Therefore, the steam is *superheated at the state 2.*

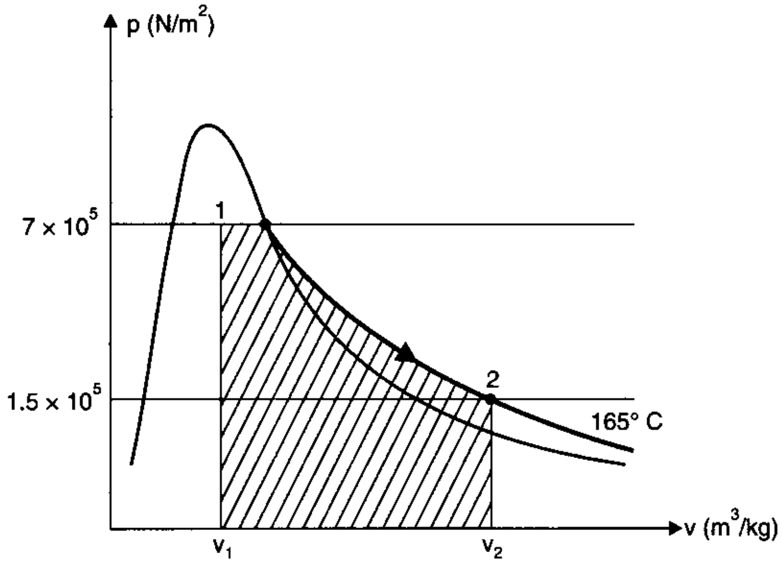


Fig. 4.63

(*i*) **Change of internal energy :**

The internal energy at state 1 is found by using the relation :

$$u_1 = (1 - x)\, u_f + x u_g$$
$$= (1 - 0.95)\, 696 + (0.95 \times 2573)$$
$$\therefore \quad u_1 = 2479.15 \text{ kJ/kg}$$

Interpolating from superheat tables at 1.5 bar and 165°C, we have

$$u_2 = 2580 + \frac{15}{50}\,(2856 - 2580)$$
$$= 2602.8 \text{ kJ/kg}$$

∴ **Gain in internal energy,**

$$u_2 - u_1 = 2602.8 - 2479.15 = \mathbf{123.65 \text{ kJ/kg.}} \quad \textbf{(Ans.)}$$

(*ii*) **Change of enthalpy :**

Enthalpy at state 1 (7 bar),

$$h_1 = h_{f_1} + x_1 h_{fg_1}$$

At 7 bar. $h_f = 697.1$ kJ/kg and $h_{fg} = 2064.9$ kJ/kg

$$\therefore \quad h_1 = 697.1 + 0.95 \times 2064.9 = 2658.75 \text{ kJ/kg}$$

Interpolating from superheat tables at 1.5 bar and 165°C, we have

$$h_2 = 2772.6 + \frac{15}{50}\,(2872.9 - 2772.6) = 2802.69 \text{ kJ/kg}$$

∴ Change of enthalpy

$$= h_2 - h_1 = 2802.69 - 2658.75 = \mathbf{143.94 \text{ kJ/kg.}} \quad \textbf{(Ans.)}$$

(*iii*) **Work done :**

From non-flow energy equation,

$$Q = (u_2 - u_1) + W$$

$$\therefore \quad W = Q - (u_2 - u_1) = 420 - 123.65 = 296.35 \text{ kJ/kg}$$

i.e., **Work done by the steam = 296.35 kJ/kg. (Ans.)**

Note. The work done is also given by the area on the Fig. 4.60 $\left(\int_{v_1}^{v_2} p dv\right)$, this can only be evaluated graphically.

Example 4.55. *In a steam engine cylinder the steam expands from 5.5 bar to 0.75 bar according to a hyperbolic law, pv = constant. If the steam is initially dry and saturated, calculate per kg of steam :*

(*i*) *Work done ;*

(*ii*) *Heat flow to or from the cylinder walls.*

Solution. Initial pressure of steam, p_1 = 5.5 bar = 5.5 × 10^5 N/m^2

Initial condition of steam, $x_1 = 1$

Final pressure of steam, p_2 = 0.75 bar = 0.75 bar × 10^5 N/m^2

At 5.5 bar, $v_1 = v_g = 0.3427$ m^3/kg

Also $p_1v_1 = p_2v_2$

$$\therefore \quad v_2 = \frac{p_1v_1}{p_2} = \frac{5.5 \times 0.3427}{0.75} = 2.513 \text{ m}^3\text{/kg}$$

At 0.75 bar, $v_g = 2.217$ m^3/kg.

Since $v_2 > v_g$ (at 0.75 bar), therefore, the steam is *superheated* at state 2.

Interpolating from superheat tables at 0.75 bar, we have

$$u_2 = 2510 + \left(\frac{2.513 - 2.271}{2.588 - 2.271}\right)(2585 - 2510)$$

$$= 2510 + \frac{0.242}{0.317} \times 75 = 2567.25 \text{ kJ/kg.}$$

For dry saturated steam at 5.5 bar

$$u_1 = u_g = 2565 \text{ kJ/kg}$$

Hence, gain in internal energy

$$= u_2 - u_1 = 2567.25 - 2565 = 2.25 \text{ kJ/kg}$$

The process is shown on a *p-v* diagram in Fig. 4.64, the shaded area representing the work done.

Now,

$$W = \int_{v_1}^{v_2} p\, dv$$

$$= \int_{v_1}^{v_2} \left(\frac{\text{constant}}{v}\right) dv \qquad \left[\because \; pv = \text{constant, and } p = \frac{\text{constant}}{v}\right]$$

$$= \text{constant}\left[\log_e v\right]_{v_1}^{v_2}$$

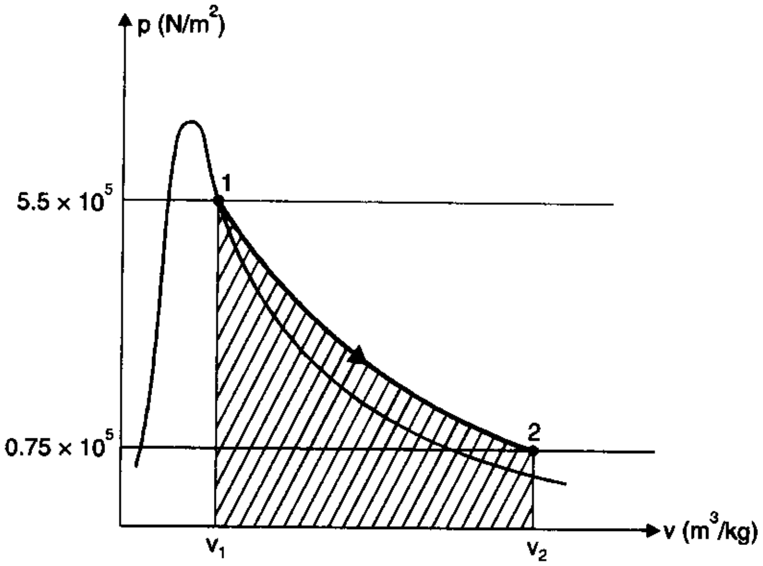


Fig. 4.64

The constant is either p_1v_1 or p_2v_2

i.e.,
$$W = 5.5 \times 10^5 \times 0.3427 \times \log_e \frac{p_1}{p_2} \qquad \left[\because\ p_1v_1 = p_2v_2 \text{ or } \frac{v_2}{v_1} = \frac{p_1}{p_2}\right]$$

$$= 5.5 \times 10^5 \times 0.3427 \times \log_e \left(\frac{5.5}{0.75}\right) = 375543 \text{ N-m/kg.}$$

Using non-flow energy equation, we get

$$Q = (u_2 - u_1) + W$$

$$= 2.25 + \frac{375543}{10^3} = 378 \text{ kJ/kg}$$

i.e., **Heat supplied** **= 378 kJ/kg. (Ans.)**

Example 4.56. *Dry saturated steam at 100 bar expands isothermally and reversibly to a pressure of 10 bar. Calculate per kg of steam :*

(i) The heat supplied ;

(ii) The work done.

Solution. Initial pressure of steam, p_1 = 100 bar

Final pressure of steam, p_2 = 10 bar

The process is shown in Fig. 4.65, the shaded area representing the heat supplied.

At 100 bar, dry saturated : From steam tables,

$$s_1 = s_g = 5.619 \text{ kJ/kg K and } t_{s_1} = 311°\text{C}$$

At 10 bar and 311°C the steam is superheated, hence interpolating

$$s_2 = 7.124 + \left(\frac{311-300}{350-300}\right)(7.301 - 7.124) \quad \text{or} \quad s_2 = 7.163 \text{ kJ/kg K.}$$

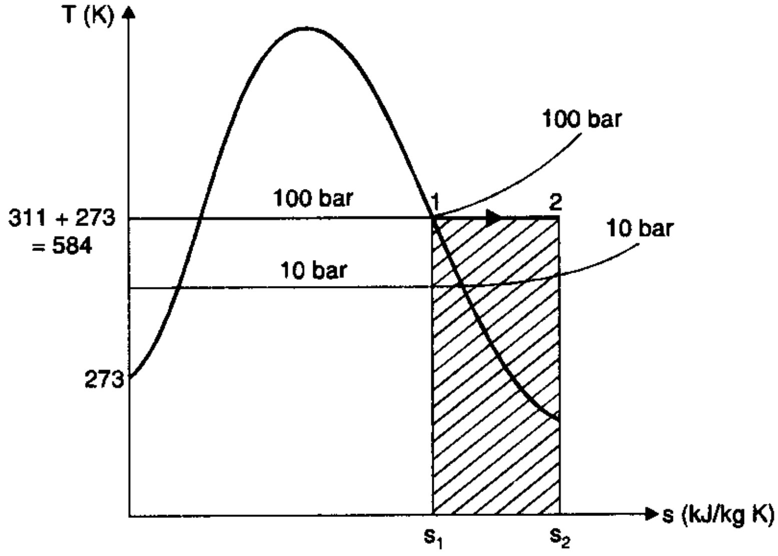


Fig. 4.65

(*i*) **Heat supplied :**

Now, heat supplied, Q = shaded area = $T(s_2 - s_1)$

$$= 584(7.163 - 5.619) = \mathbf{901.7\ kJ/kg.}\ \ \textbf{(Ans.)}$$

(*ii*) **Work done :**

To find work done, applying non-flow energy equation,

$$Q = (u_2 - u_1) + W$$

or $$W = Q - (u_2 - u_1)$$

From steam tables at 100 bar, dry saturated,

$$u_1 = u_g = 2545 \text{ kJ/kg}$$

At 10 bar 311°C, interpolating,

$$u_2 = 2794 + \left(\frac{311-300}{350-300}\right)(2875 - 2794)$$

i.e., $$u_2 = 2811.8 \text{ kJ/kg}$$

Then, $$W = Q - (u_2 - u_1)$$
$$= 901.7 - (2811.8 - 2545) = 634.9 \text{ kJ/kg}$$

Hence, **work done by the steam = 634.9 kJ/kg. (Ans.)**

4. Reversible Adiabatic or Isentropic Process. Fig. 4.66 (*a*), (*b*) and (*c*) shows the isentropic process on *p-v*, *T-s* and *h-s* diagrams respectively.

Let us consider that the process is non-flow reversible adiabatic. Now applying first law energy equation, we have

$$Q = \Delta u + \int_1^2 p\,.dv = (u_2 - u_1) + W$$

As for adiabatic process, $Q = 0$

$$\therefore \quad W = (u_1 - u_2) \qquad \text{...(4.77)}$$

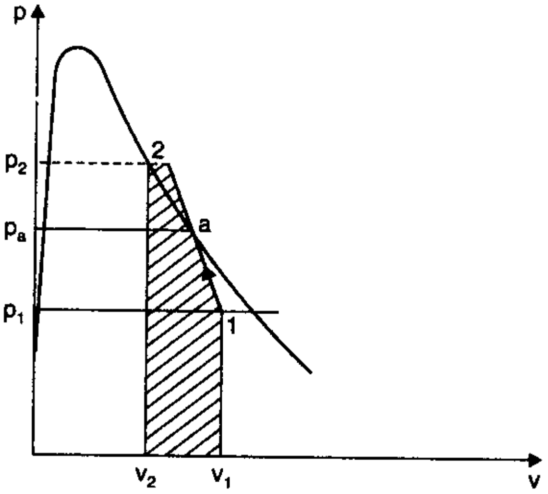
In case the process is steady flow reversible adiabatic, then first law energy equation can be written as

$$u_1 + p_1v_1 + Q = u_2 + p_2v_2 + W$$

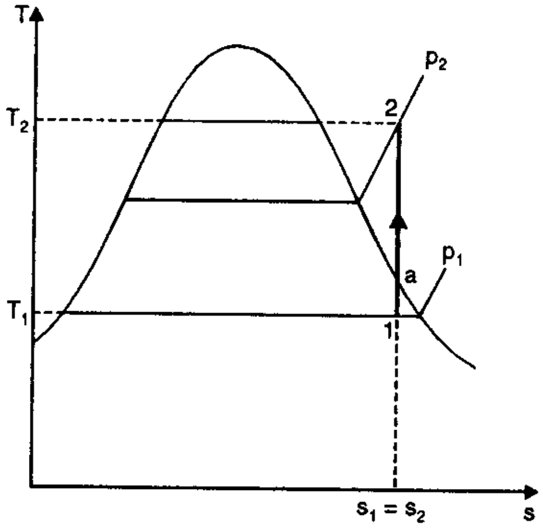
where v_1 and v_2 are the specific volumes of steam before and after executing the process.

$$\therefore \quad h_1 + 0 = h_2 + W \qquad (\because \quad Q = 0)$$

$$\therefore \quad W = (h_1 - h_2) \qquad ...(4.78)$$



(*a*)



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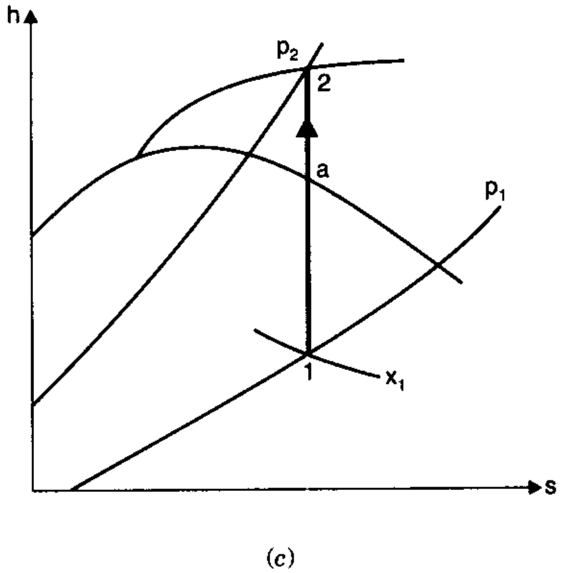


(c)

Fig. 4.66. Reversible adiabatic or isentropic process.

☞**Example 4.57.** *1 kg of steam at 120 bar and 400°C expands reversibly in a perfectly thermally insulated cylinder behind a piston until the pressure is 38 bar and the steam is then dry saturated. Calculate the work done by the steam.*

Solution. Mass of steam, $m = 1$ kg

Initial pressure of steam, $p_1 = 120$ bar $= 120 \times 10^5$ N/m^2

Initial temperature of steam, $t_1 = 400$°C.

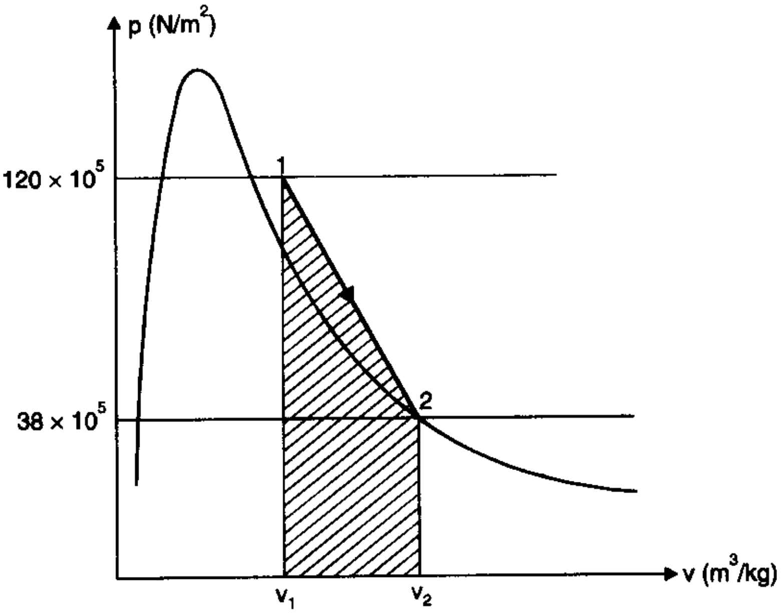


Fig. 4.67

Final pressure of steam, $p_2 = 38$ bar

From superheat tables, at *120 bar and 400°C*

$h_1 = 3051.3$ kJ/kg and $v_1 = 0.02108$ m^3/kg

Now, using the equation :

$$u = h - pv$$

$$\therefore \quad u_1 = 3051.3 - \frac{120 \times 10^5 \times 0.02108}{10^3} = 2798.34 \text{ kJ/kg}$$

Also, $\quad u_1 = u_g$ at 38 bar = 2602 kJ/kg.

Since the cylinder is perfectly thermally insulated then no heat flows to or from the steam during the expansion, the process therefore is adiabatic.

∴ **Work done by the steam,** $W = u_1 - u_2$

$= 2798.34 - 2602 =$ **196.34 kJ/kg. (Ans.)**

The process is shown on *p-v* diagram in Fig. 4.67, the shaded area representing the work done.

5. Polytropic process. In this process, the steam follows the law pv^n = constant. This process on *p-v*, *T-s* and *h-s* diagrams is shown in Fig. 4.68 (*a*), (*b*) and (*c*).

The work done during this process is given by

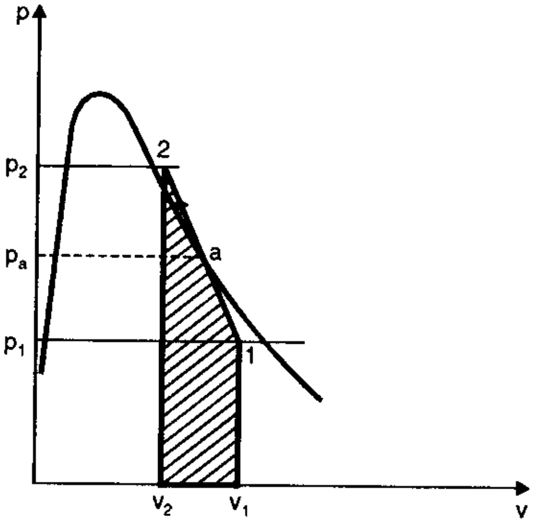
$$W = \left(\frac{p_1v_1 - p_2v_2}{n-1}\right) \text{ N-m/kg}$$

Applying the first law energy equation to non-flow process, we have

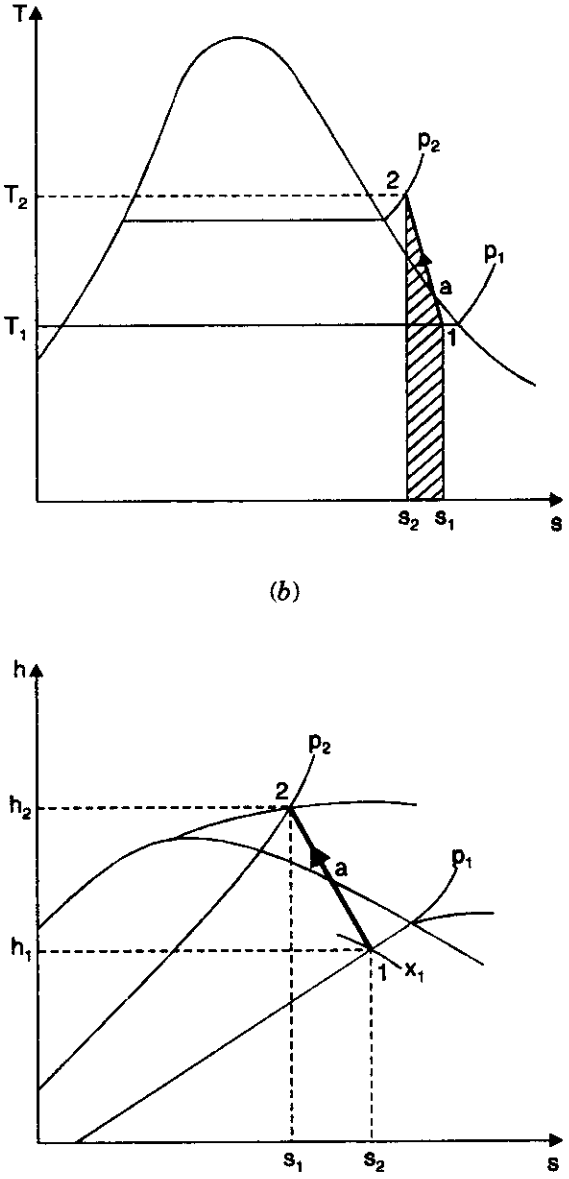
$$Q = \Delta u + W$$

$$= (u_2 - u_1) + \left(\frac{p_1v_1 - p_2v_2}{n-1}\right)$$

$$= (h_2 - p_2v_2) - (h_1 - p_1v_1) + \left(\frac{p_1v_1 - p_2v_2}{n-1}\right)$$



(*a*)



(b)

(c)

Fig. 4.68. Polytropic process.

$$= (h_2 - h_1) + (p_1v_1 - p_2v_2)\left(1 + \frac{1}{n-1}\right)$$

$$= (h_2 - h_1) + \frac{n}{n-1}(p_1v_1 - p_2v_2) \qquad ...(4.79)$$

In adiabatic process $Q = 0$ and if $\Delta s \neq 0$ then the process behaves like *adiabatic process* and *not isentropic*. Such a process with steam will be a particular case of the law pv^n = constant. The index n in this case will be that particular index which will satisfy the condition :